THEORETICAL INVESTIGATION OF THE EFFECT OF PARTICLE CONTAMINANTS ON LASER-INDUCED AIR BREAKDOWN

Frank D. Feiock, et al

KMS Technology Center

Prepared for:

Air Force Weapons Laboratory
Advanced Research Projects Agency

April 1973

AFWL-TR-72-172

AFWL-TR
72-172

THEORETICAL INVESTIGATION OF THE EFFECT OF PARTICLE CONTAMINANTS ON LASER-INDUCED AIR BREAKDOWN

Frank D. Feiock

Lester K. Goodwin

KMS Technology Center

TECHNICAL REPORT NO. AFWL-TR-72-172

April 1973

AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base
New Mexico

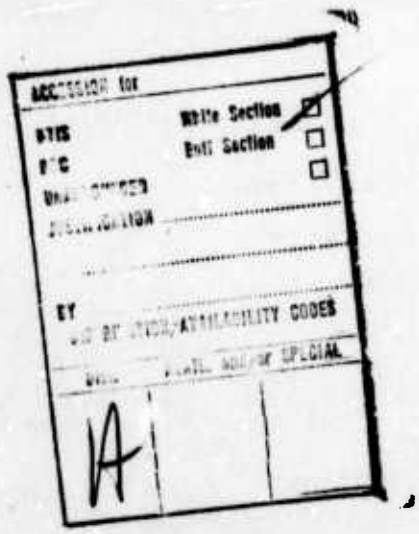
Approved for public release; distribution unlimited.

AFWL-TR-72-172

AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base
New Mexico 87117

When US Government drawings, specifications, or other data are used for any purpose other than a definitely related Government procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise, as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

DO NOT RETURN THIS COPY. RETAIN OR DESTROY.



UNCLASSIFIED
Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) KMS Technology Center Irvine, California 92664	2a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED 2b. GROUP

3. REPORT TITLE

THEORETICAL INVESTIGATION OF THE EFFECT OF PARTICLE CONTAMINANTS ON LASER-INDUCED AIR BREAKDOWN

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
22 June 1971-30 August 1972

5. AUTHOR(S) (First name, middle initial, last name)
Frank D. Feiock; Lester K. Goodwin

6. REPORT DATE April 1973	7a. TOTAL NO. OF PAGES ~~194~~ 196	7b. NO. OF REFS 45

8a. CONTRACT OR GRANT NO. F29601-71-C-0118 b. PROJECT NO. 1256 c. Task No. 09 d.	9a. ORIGINATOR'S REPORT NUMBER(S) AFWL-TR-72-172 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Approved for public release; distribution unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY AFWL (DYT) Kirtland AFB, NM 87117

13. ABSTRACT

(Distribution Limitation Statement A)

The effect of atmospheric contaminants on the laser-induced breakdown of air is investigated. Calculations were performed using a one-dimensional, Lagrangian, hydrodynamic code devieloped for ~~this~~ study. The results of this study indicate that the interaction of a laser beam with atmospheric contaminants such as SiO_2 and Al_2O_3 can supply a primary electron density that will lead to a significant decrease in the laser-induced breakdown threshold for air.

DD FORM 1 NOV 68 1473

UNCLASSIFIED
Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Laser-induced gas breakdown						
Atmospheric particles						
Two-temperature hydrodynamics						
CO_2 laser						
Thermionic emission						

AFWL-TR-72-172

THEORETICAL INVESTIGATION OF THE EFFECT OF PARTICLE CONTAMINANTS ON LASER-INDUCED AIR BREAKDOWN

Frank D. Feiock
Lester K. Goodwin
KMS Technology Center

TECHNICAL REPORT NO. AFWL-TR-72-172

FOREWORD

This report was prepared by KMS Technology Center, Irvine, California, under Contract F29601-71-C-0118. The research was performed under Program Element 62301D, Project 1256, Task 09, and was funded by the Advanced Research Projects Agency (ARPA) under ARPA Order 1256.

Inclusive dates of research were 22 June 1971 through 30 August 1972. This report was submitted 1 March 1972 by the Air Force Weapons Laboratory Project Officer, Captain Philip E. Nielsen (DYT).

Publication of this report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

Philip E. Nielsen

PHILIP E. NIELSEN
Captain, USAF
Project Officer

JAY R. ROLAND
Major, USAF
Chief, Theoretical Physics Branch

JOHN K. LEROHL
Colonel, USAF
Chief, Technology Division

CONTENTS

CONTENTS (Cont.)

ILLUSTRATIONS

Illustrations (Cont.)

SECTION I

INTRODUCTION

The optical properties of a medium can be altered by a laser beam of sufficient intensity. This alteration can range from relatively small changes in the index of refraction leading to diffraction effects to complete breakdown of the propagating medium. Thus, the interaction between a laser beam and the surrounding environment can impose limitations on the size and intensity of the beam that is to be used for a given purpose.

In the case of air, laboratory experiments with the 10.6 μ radiation from a CO_2 laser have shown that atmospheric breakdown occurs at an intensity of approximately 2×10^9 watts/cm^2 for sea level density (Ref. 1). It is shown in Section II that this is about the same intensity predicted by theoretical considerations if it is assumed that initially there is one free electron per cm^3.

Further experiments indicated that the breakdown threshold for air could be as low as 10^8 watts/cm^2 (Ref. 2). Since the later experiments were carried out with air under normal conditions while the earlier experiments used cleaner air meeting laboratory standards, it was postulated that the lowering of the breakdown threshold was due to the presence of an increased amount of contaminants in the dirty air.

Contaminants such as quartz (SiO_2) and sapphire (Al_2O_3) are assumed to be present in the form of solid particles ranging in size from 1 μ to 100 μ radius. These particles absorb energy, finally reaching the vaporization temperature and moving out into the air. Because of the higher absorption of energy the contaminants can provide the initial source of electrons leading to atmospheric breakdown.

The purpose of the present study is to develop a model suitable for

investigation of the effects of contaminants on atmospheric breakdown. This model utilizes where possible, the known material properties of the contaminant particle to determine the absorption of the laser energy and the subsequent heating. A one dimensional Lagrangian hydrodynamic code, CELAB (Contaminant Effects on Laser Air Breakdown), has been developed to compute the motion of the vaporized material.

Semi-emperical expressions for the relevant molecular cross sections are developed to describe the interactions between the free electrons from the contaminant particle and the atmosphere. The calculated values for the cross sections are compared with the latest data available and found to give reasonable results over a wide range of electron energy.

The results of preliminary calculations on quartz (SiO_2) and sapphire (Al_2O_3) particles of varying sizes indicate that atmospheric contaminants are an important factor in the laser induced breakdown of air. First, they supply a substantial density of free electrons for the coupling of laser energy. Secondly, because of the relatively high density of the solid contaminant, the local degree of ionization in the neighborhood of the evaporated particle will be high making the electron-ion interaction the dominant elastic collision process. This, in turn, increases the rate at which an electron gains energy from the laser and decreases the breakdown threshold intensity.

SECTION II

AIR BREAKDOWN ANALYSIS

The intensity of high-power laser beams propagating through the atmosphere is limited by atmospheric breakdown. For the 10.6-micron radiation of the CO_2 laser, experiments on ordinary laboratory air show that breakdown is initiated at laser powers of about 2×10^9 watts/cm^2. If it is assumed that there is an initial density of free electrons in the focal volume of the laser beam then the breakdown process can be described by electron cascade theory (Ref. 3). In this theory, the free electrons in the focal volume of the beam absorb energy by inverse bremsstrahlung. When the rate of energy absorption is large enough for an electron to reach the ionization energy during the laser pulse duration, there follows a net rate of growth for the free electron population. If this growth rate exceeds the rate of loss of electrons to attachment, diffusion, etc., then the electron density increases with increasing time and breakdown of the gas is taking place. The breakdown of the gas is complete when the growth rate is large enough that complete ionization takes place within the time of the pulse duration. The process can be described by the rate equation,

$$N_e(t) = N_e(0)e^{\int_0^t (\bar{\gamma} - \alpha_L)dt} \qquad (1)$$

where $N_e(t)$ is the time dependent electron density, $\bar{\gamma}$ is the growth rate, and α_L is the loss rate. Assuming that the rates are constant over the duration of the laser pulse, the criterion for breakdown requires that the growth rate satisfies the condition

$$\bar{\gamma} > \alpha_L \qquad (2)$$

At standard temperature and pressure conditions (STP) in air, complete breakdown requires

$$\bar{\gamma} = \alpha_L + \frac{45}{t_p} \tag{3}$$

where it has been assumed that a nominal value of $N_e(0) = 1\ cm^{-3}$ exists and $N_e(t_p) = 3 \times 10^{19}\ cm^{-3}$. In the remainder of the report, the term "breakdown" will always refer to complete breakdown unless otherwise specified.

The dominant loss mechanism in air is 3-body attachment to molecular oxygen,

$$O_2 + M + e \rightleftarrows O_2^- + M \tag{4}$$

where M is a third body. The rate for this process is (Ref. 4)

$$\alpha_L \sim 7.5 \times 10^7 (\rho/\rho_o)^2\ sec^{-1} \tag{5}$$

where ρ_o is the sea level density. At STP conditions,

$$\bar{\gamma} \approx 7.5 \times 10^7 + \frac{45}{t_p}\ sec^{-1} \tag{6}$$

Thus, for very long pulses, the threshold growth rate for breakdown is

$$\bar{\gamma} = 7.5 \times 10^7\ sec^{-1} \tag{7}$$

while for pulses in the nanosecond range

$$\bar{\gamma} \approx \frac{45}{t_p}\ sec^{-1} \tag{8}$$

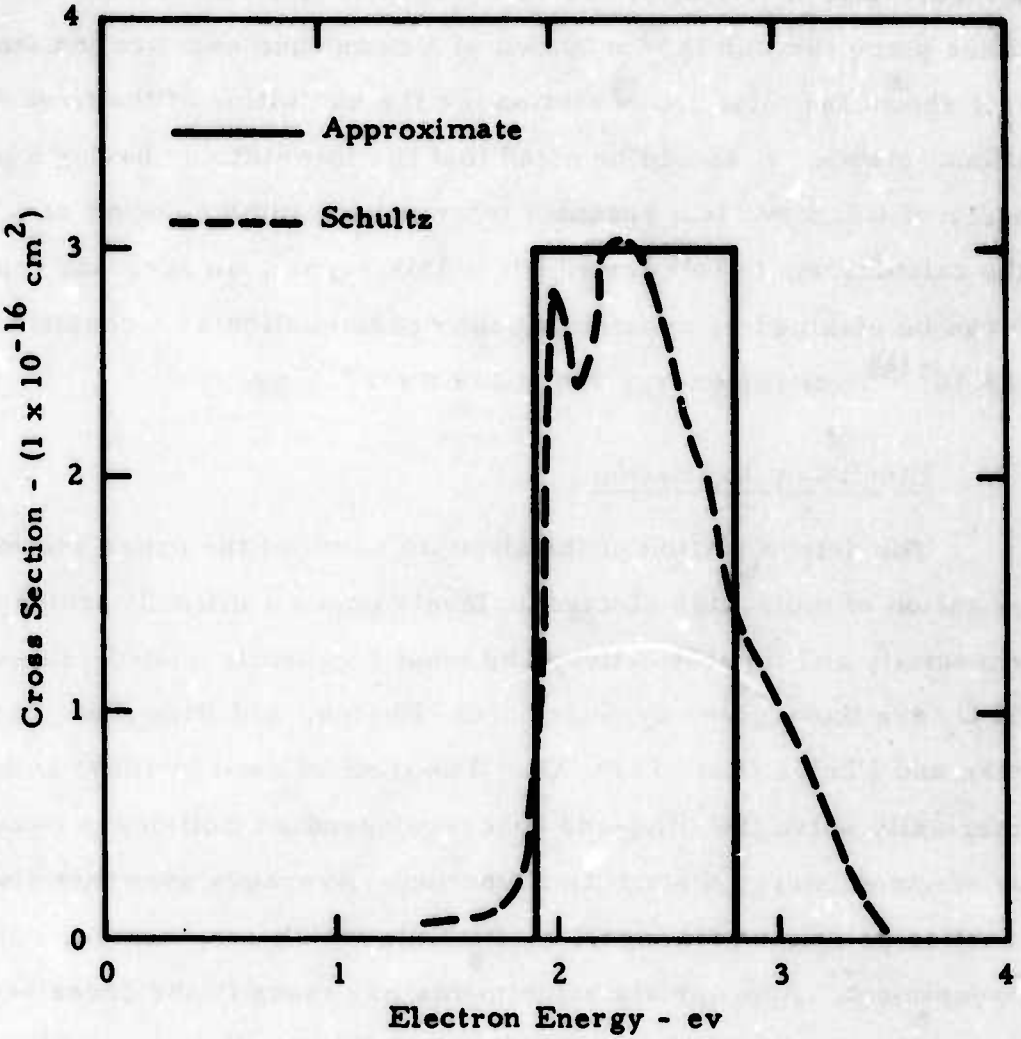


Figure 1. Sum of N_2 Vibrational Cross Sections

1. CROSS SECTIONS

a. Vibrational Excitation

The vibrational excitation of N_2 by electron impact has been studied quite extensively both experimentally and theoretically (Ref. 5). In the theoretical analysis, it has been assumed that the vibrational excitation takes place through the formation of a compound negative ion state. Figure 1 shows the total cross section for the excitation of the first eight vibrational states. It should be noted that the interaction, having a peak at an energy of ~ 2.3 ev, is a resonant interaction having a cut off at ~ 3.5 ev. For the calculations to be carried out in this report, an adequate representation can be obtained by considering the cross section as a constant $\sigma_v \sim 3 \times 10^{-16}$ over the energy range $1.9 \leq \epsilon \leq 2.8$ ev.

b. Electronic Excitation

The determination of the absolute value of the cross sections for the excitation of molecular electronic levels poses a difficult problem both experimentally and theoretically. The most frequently quoted values for N_2 and O_2 are those given by Engelhardt, Phelps, and Risk (Ref. 6) for N_2 and Hake and Phelps (Ref. 7) for O_2. The method used by these authors is to numerically solve the time-and space-independent Boltzmann equation for the electron energy distribution function. Averages over this distribution function determine transport coefficients which are, in turn, compared with experiment. Appropriate adjustments are made to the cross sections until a satisfactory fit with the experimental data for the transport coefficients are obtained. The thresholds for the various cross sections are obtained from other experimental data. The cross sections giving the best fit to the experimental transport coefficients over a wide energy range are then plotted.

More recently, Borst (Ref. 8) has made an experimental determination of the absolute cross sections for the excitation of the $A^3\Sigma_u^+$,

$a\,^1\Pi_g$ and $E\,^3\Sigma_g^+$ states of N_2. Ajello (Ref. 9) has also made measurements of the cross section for the excitation of the $a\,^1\Pi_g$ state of N_2. There appears to be substantial disagreement between the two measurements. The experimental data will be shown in appropriate portions of this section where comparisons are to be made with calculated values of the cross sections.

An approximate method for the calculation of the relevant cross sections can be obtained from classical mechanics. The transfer of energy between two electrons is

$$\Delta\epsilon = \epsilon \sin^2 \theta/2 \tag{9}$$

where ϵ is the kinetic energy of the incident electron and θ is the scattering angle. The relation between the impact parameter, energy, and scattering angle is (Ref. 10)

$$b = \frac{e^2}{2\epsilon} \cot \theta/2 \tag{10}$$

hence,

$$\Delta\epsilon = \frac{\epsilon}{1 + \frac{4b^2\epsilon^2}{e^4}} \tag{11}$$

where b is the impact parameter and e is the electronic charge.

According to classical mechanics, the cross section is defined by the integral

$$\sigma(\epsilon) = 2\pi \int_0^{b_M} b\,db = \pi b_M^2 \tag{12}$$

where b_M is the maximum value of the impact parameter for the particular interaction process under consideration. For electronic excitation, the

required value of the impact parameter can be determined from equation (11) by setting $\Delta\epsilon = \epsilon_x$,

$$b_M^2 = \frac{e^4}{4\epsilon^2 \epsilon_x} (\epsilon - \epsilon_x) \tag{13}$$

and

$$\sigma_x(\epsilon) = \frac{\pi e^4}{4\epsilon^2 \epsilon_x} (\epsilon - \epsilon_x) \tag{14}$$

where ϵ_x is the excitation energy. Equation (14) has a peak value at $\epsilon = 2\epsilon_x$ giving a peak value of the cross section

$$\sigma_x(2\epsilon_x) = \frac{\pi e^4}{16\epsilon_x^2} \tag{15}$$

Equation (14) seems to give reasonable behavior for singlet-singlet transitions such as $^1\Sigma_g^+ \rightarrow a\,^1\Pi_g$. However, for the singlet-triplet transitions such as $^1\Sigma_g^+ \rightarrow A\,^3\Sigma_u^+$, equation (14) does not fall off rapidly enough with increasing energy. The functional form for the triplet excitation cross section is

$$\sigma_x(\epsilon) = \sigma_1 [a(\epsilon - \epsilon_x)]\, e^{-a\epsilon} \tag{16}$$

where σ_1 and a are constants to be determined.

The maximum value for the cross section given by equation (16) occurs at the energy

$$\epsilon_M = \frac{1}{a} + \epsilon_x$$

Setting $\epsilon_M = 2\epsilon_x$ so that the peak cross section occurs at the same energy as predicted by equation (14) gives the result $a = \epsilon_x^{-1}$, and

$$\sigma_x(\epsilon) = \sigma_1 \left(\frac{\epsilon - \epsilon_x}{\epsilon_x} \right) e^{-\epsilon/\epsilon_x} \tag{17}$$

Setting $\epsilon = 2\epsilon_x$ in equation (17) and equating the resulting cross section to the peak cross section given by equation (14) leads to an expression for σ_1,

$$\sigma_1 = 0.461 \frac{\pi e^4}{\epsilon_x^2} \tag{18}$$

and

$$\sigma_x(\epsilon) = 0.461 \frac{\pi e^4}{\epsilon_x^3} \left(\epsilon - \epsilon_x \right) e^{-\epsilon/\epsilon_x} \tag{19}$$

Equations (14) and (19) will be used to calculate the various electronic excitation processes of interest in this report and the results compared with existing experimental data and the results of other calculations.

(1) Nitrogen Cross Section for $A^3\Sigma_u^+$ State

The excitation energy for this state is about $\epsilon_x = 6.7$ ev (Ref. 11); hence, the calculated peak value of the cross section will occur at $\epsilon = 13.4$ ev and the peak value of the cross section calculated from equation (19) is $\sigma_x(2\epsilon_x) = 9.1 \times 10^{-17}\,\mathrm{cm}^2$.

The experimental data of Borst (Ref. 8) place the peak at $\epsilon \sim 11$ ev, which is in reasonable agreement with calculated value. The measured value of the peak cross section is $\sigma_x = \left(5.3^{+4.0}_{-3.0}\right) \times 10^{-17}\,\mathrm{cm}^2$. The upper limit is very close to the calculated value.

The calculated peak value of Engelhardt, et al. (Ref. 6) is $\sigma_x \approx 0.6 \times 10^{-16} cm^2$, but their peak occurs at about 7.5 ev, which is about 4 ev lower than the experimental value. The calculated value of Chung and and Lin (Ref. 12) has a peak value at about 11 ev, but their calculated peak cross section is $\sigma_x \approx 1.21 \times 10^{-16} cm^2$.

The cross section used by Kroll and Watson (Ref. 11) for excitation of the $N_2(A^3\Sigma_u^+)$ level is

$$\sigma_x(\epsilon) = 3.45 \times 10^{-16} (\epsilon - 6.7)\, e^{-\epsilon/6.7}\ cm^2 \qquad (20)$$

where ϵ is in units of ev. This is to be compared with the equation used in this report,

$$\sigma_x(\epsilon) = 1.01 \times 10^{-16} (\epsilon - 6.7)\, e^{-\epsilon/6.7}\ cm^2 \qquad (21)$$

Thus, Kroll and Watson obtain a peak cross section about a factor of 3 higher than obtained in the present calculation. This is attributed to the fact that Kroll and Watson multiplied by a scale factor to give the measured low-frequency breakdown threshold in air.

Various excitation functions for the $A^3\Sigma_u^+$ state of N_2 are compared in figure 2. The peak value error bar given by Borst is shown to indicate the magnitude of the experimental uncertainties involved. The values of Kroll and Watson equation (20) are not plotted since they are similar to those for equation (21) except off scale a factor of about 3 higher.

(2) Nitrogen Cross Section for $B^3\Pi_g$ State

The excitation energy for this state is about 7.4 ev, giving a peak value of the cross section at 14.8 ev of $7.5 \times 10^{-17} cm^2$ calculated from equation (19). Chung and Lin (Ref. 12) report both a corrected experimental excitation function for N_2 and a theoretical one. These are

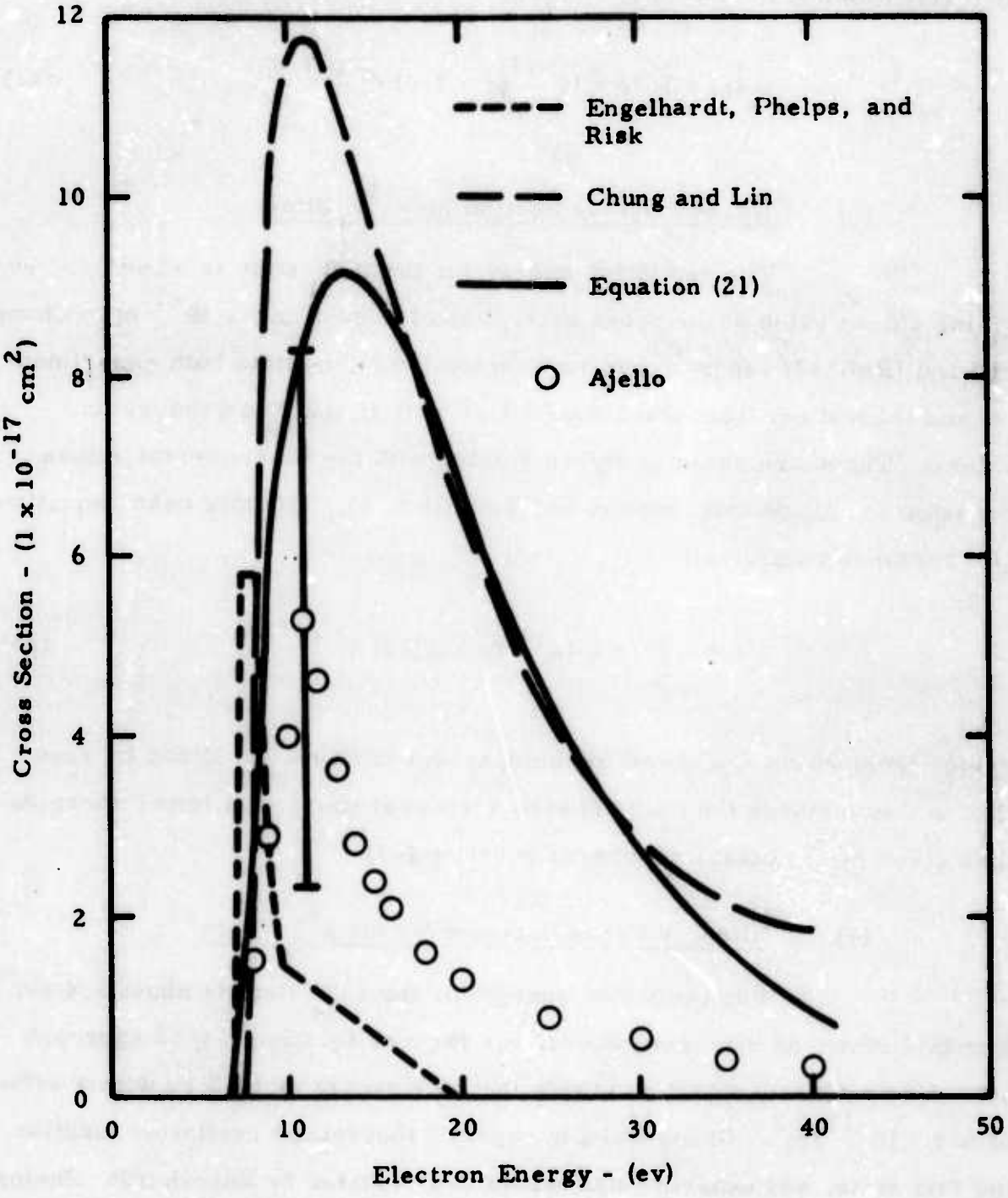


Figure 2. Excitation Functions for the $A^3\Sigma_u^+$ State For N_2

shown in figure 3. Also shown is the theoretical curve obtained from equation (19) which is numerically

$$\sigma_x(\epsilon) = 7.16 \times 10^{-17}(\epsilon - 7.4)\, e^{-\epsilon/7.4} \qquad (22)$$

(3) Nitrogen Cross Section for $C^3\Pi_u$ State

The excitation energy for the $C^3\Pi_u$ state is about 11.2 ev giving a peak value of the cross section at 22.4 ev of 3.3×10^{-17} ev. Chung and Lin (Ref. 12) report excitation functions for this state both experimental and theoretical from other sources as well as their own theoretical values. These are shown in figure 4 along with the experimental values reported by Engelhardt, Phelps and Risk (Ref. 6). For this case, equation (19) becomes numerically

$$\sigma_x(\epsilon) = 2.15 \times 10^{-17}(\epsilon - 11.2)\, e^{-\epsilon/11.2} \qquad (23)$$

values from which are shown for comparison in figure 4. It can be seen that in this instance the peak appears somewhat sharper at lower energies than given by the classical form of equation (23).

(4) Nitrogen Cross Sections for the $a^1\Pi_g$ State

The excitation energy for the $a^1\Pi_g$ state is about 8.4 ev. For this state, as discussed above, the form of equation (14) is appropriate. The peak cross section in this instance occurs at 16.8 ev with a value of 5.8×10^{-17} cm^2. Chung and Lin report a theoretical excitation function for this state, and experimental values are reported by Engelhardt, Phelps and Risk (Ref. 6), by Ajello (Ref. 9) and by Borst (Ref. 8), all of which are shown in figure 5. The numerical form of equation (14) for this case is

$$\sigma_x(\epsilon) = 1.94 \times 10^{-15}(\epsilon - 8.4)/\epsilon^2 \qquad (24)$$

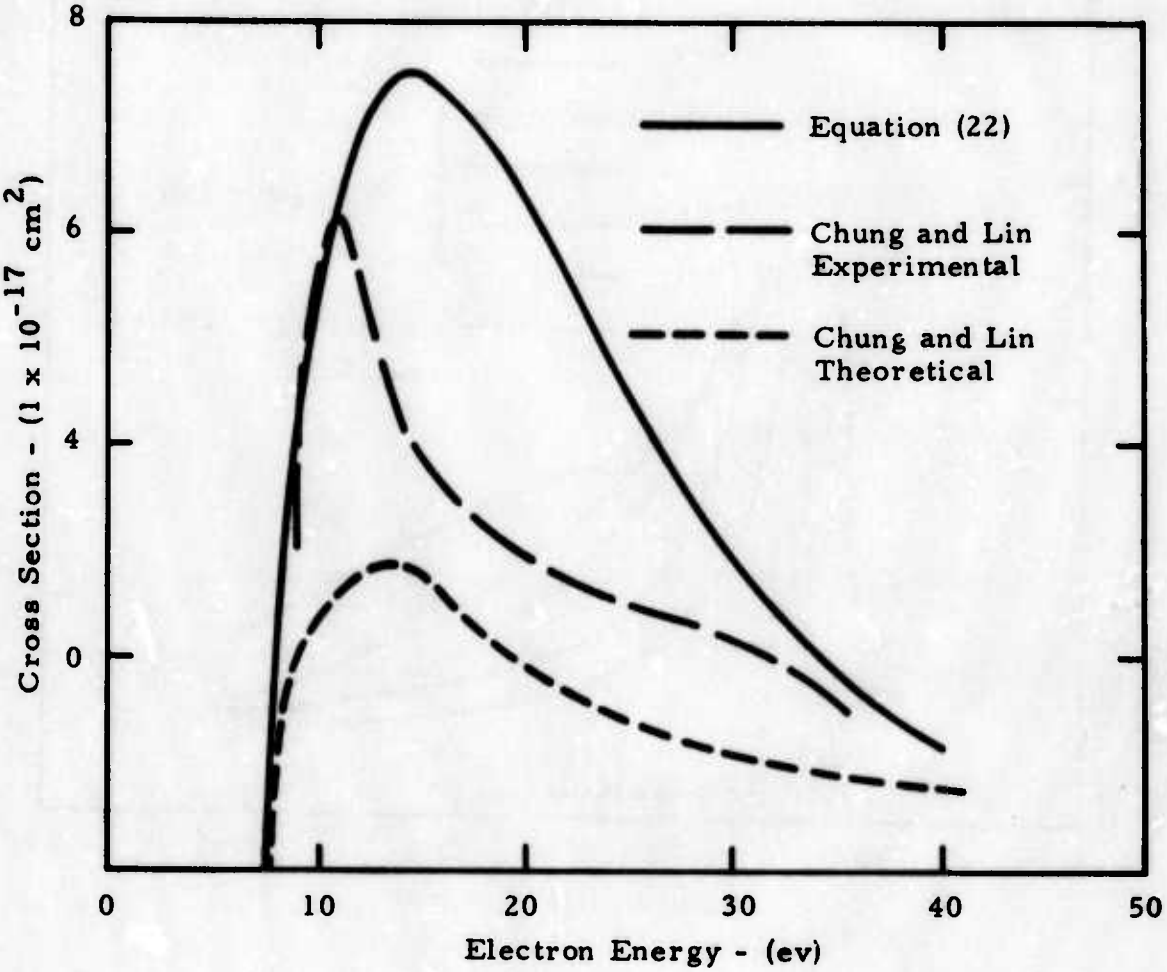


Figure 3. Excitation Functions for the $B^3\Pi_g$ State For N_2

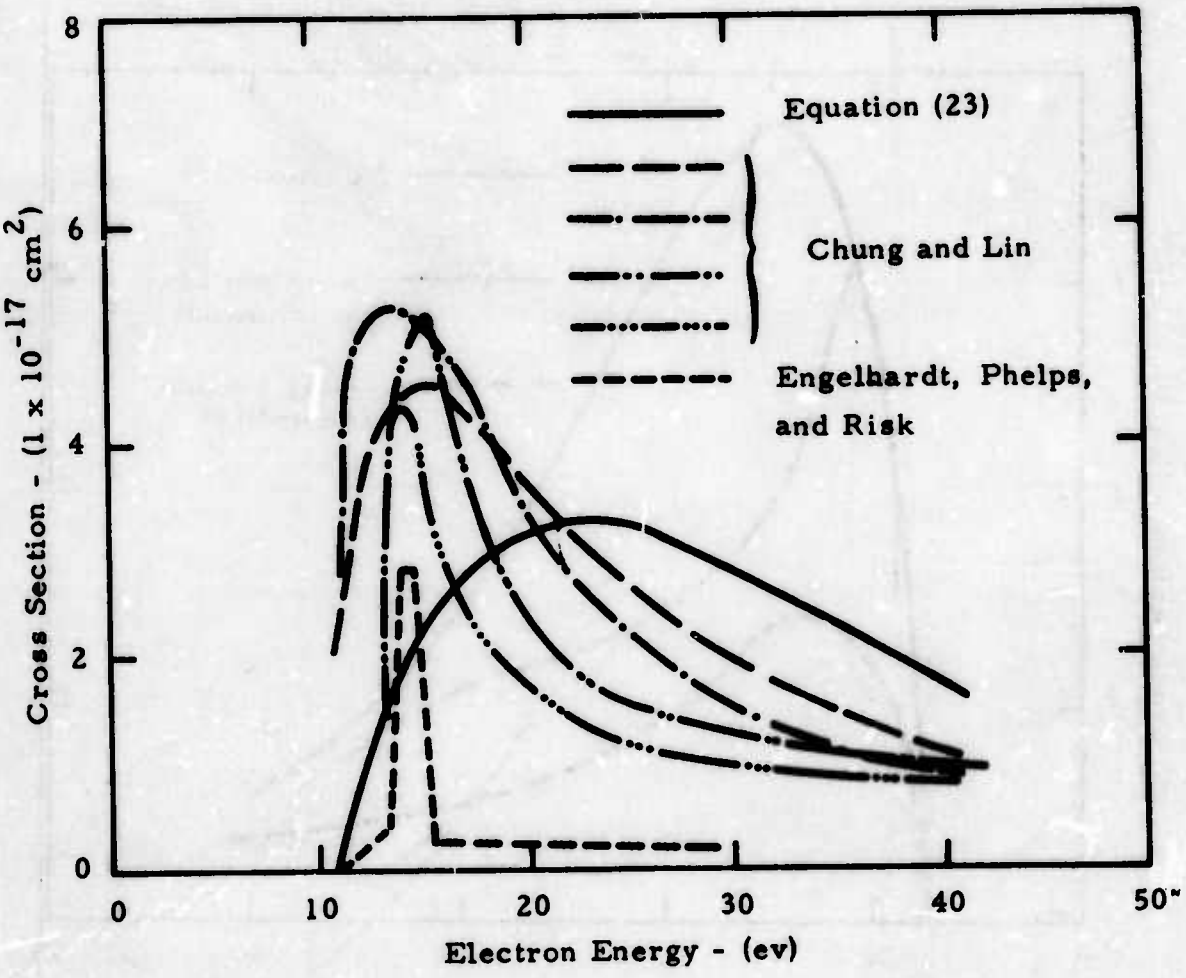


Figure 4. Excitation Functions for the $C^3\Pi_u$ State For N_2

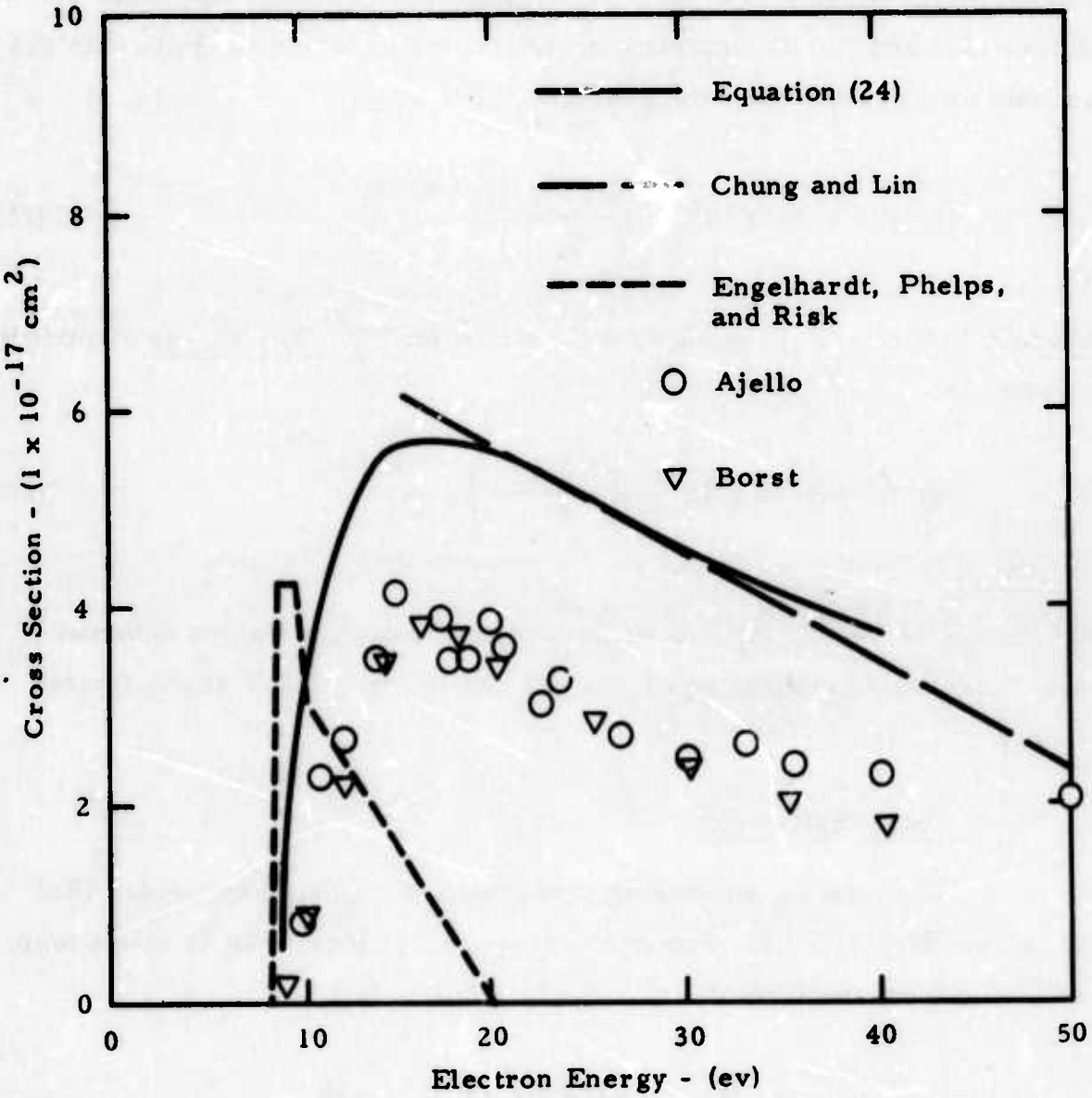


Figure 5. Excitation Functions for the $a^1\Pi_g$ State For N_2

Values obtained from this equation are also shown in figure 5 for comparisons.

c. Ionization

Extensive data exists for ionization cross sections in both nitrogen and oxygen. Values obtained from a summary of these data are shown in figures 6 and 7. Approximate empirical forms which fit these data reasonably well at least over the peak are for N_2:

$$\sigma_i(\epsilon) = 1.76 \times 10^{-15} \left(\frac{\epsilon - \epsilon_i}{\epsilon_i}\right) e^{-\epsilon/5\epsilon_i} \tag{25}$$

where $\epsilon = 15.58$ ev, the ionization threshold for N_2. For O_2 the empirical equation is:

$$\sigma_i(\epsilon) = 5.10 \times 10^{-16} \left(\frac{\epsilon - \epsilon_i}{\epsilon_i}\right) e^{-\epsilon/15\epsilon_i} \tag{26}$$

where $\epsilon_i = 12.06$ ev, the ionization threshold for O_2. Values obtained from these two empirical equations are shown for comparison in figures 6 and 7.

d. Total Scattering

The total N_2 scattering cross section reported by Golden (Ref. 13) is shown in figure 8. For comparison, an empirical fit is also shown calculated from the formula.

$$\sigma_T(\epsilon) = \left[1 + 2e^{-2(\epsilon - 2.5)^2}\right] \times 10^{-15} \tag{27}$$

All of the empirical formulae described above are approximate, but considering the large uncertainties present in much of the experimental

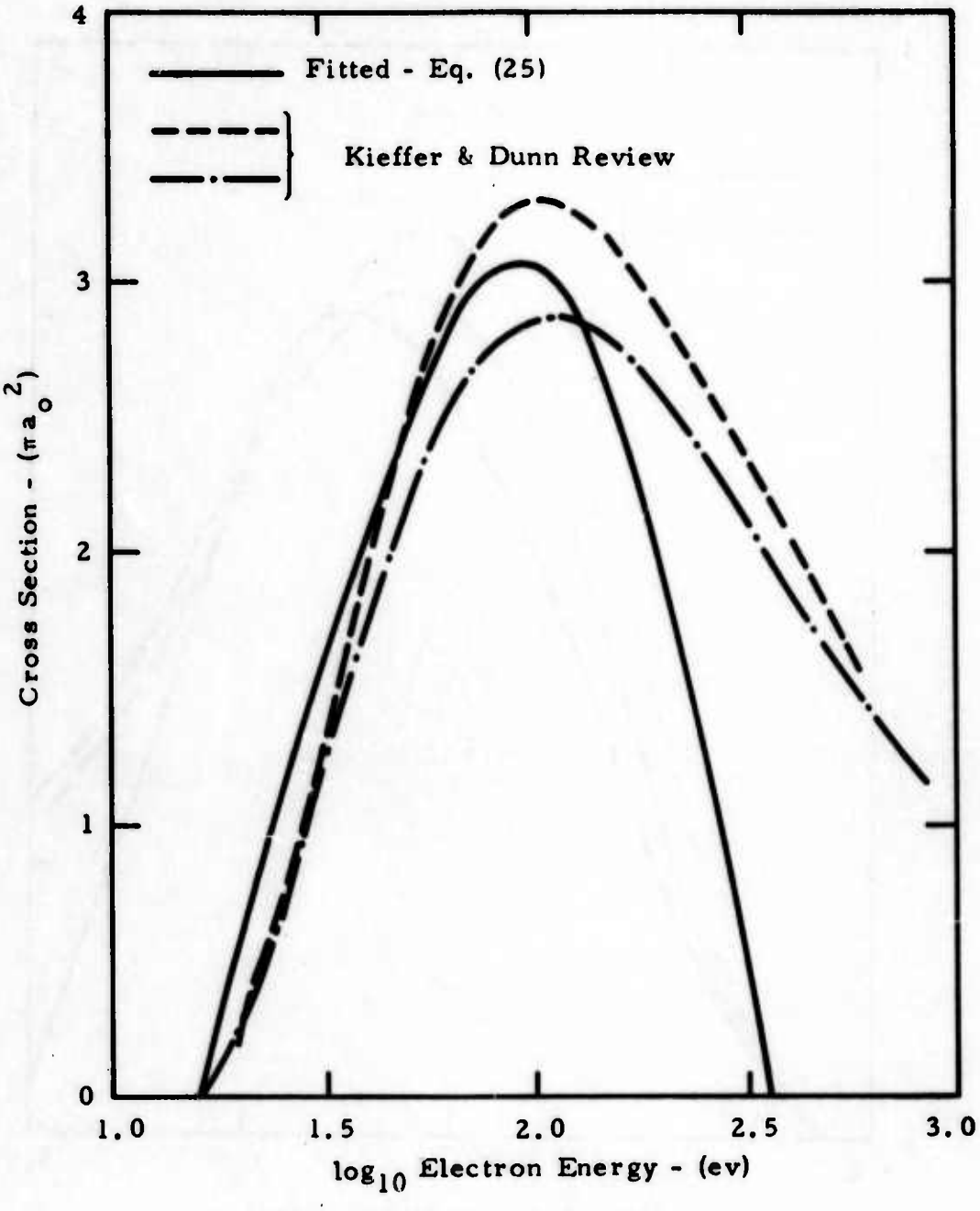


Figure 6. Total N_2 Ionization Cross Section

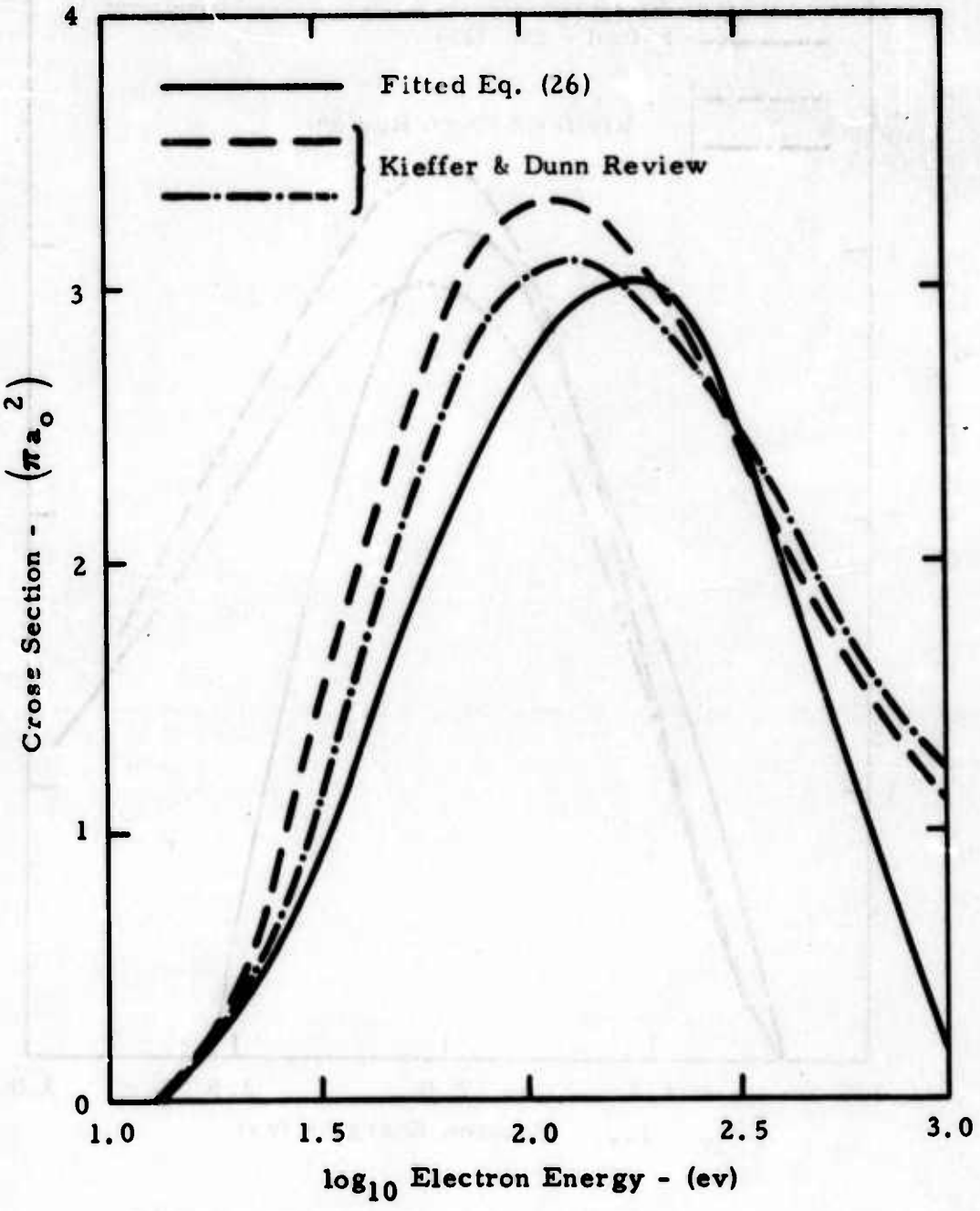


Figure 7. Total O_2 Ionization Cross Section

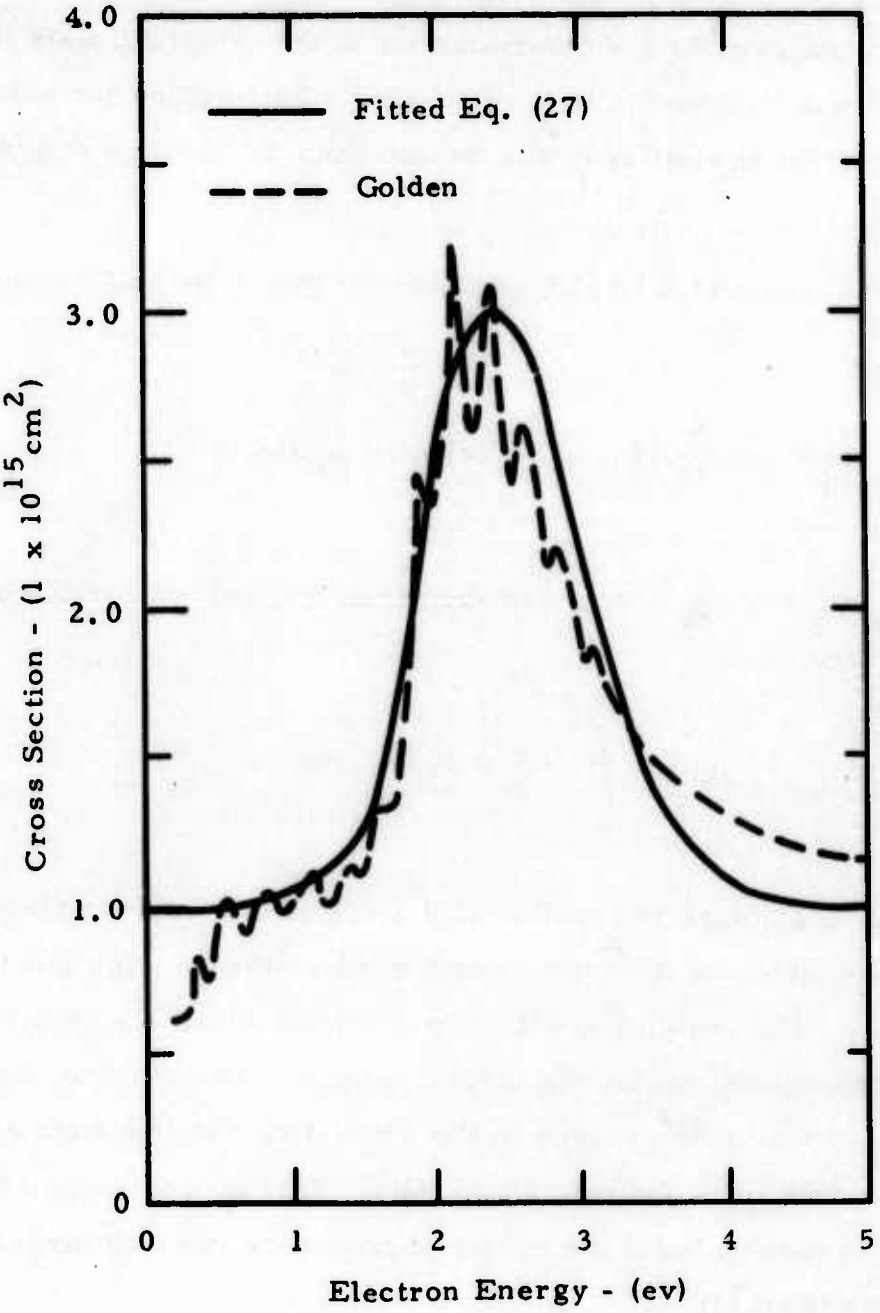


Figure 8. Total N_2 Electron Scattering Cross Section

data available, these formulae are useful for purposes of estimation.

2. BREAKDOWN ANALYSIS

An analysis for the determination of the threshold field leading to breakdown will be given in this subsection. The method for solving the kinetic equation is similar to the method used by Baraff and Buchsbaum (Ref. 14).

An approximate kinetic equation for electrons gaining energy at a rate k is

$$\left(\frac{\partial}{\partial t} + k\,\frac{\partial}{\partial \epsilon}\right) f(\epsilon, t) = \delta(\epsilon)\, g(t) - \nu_{xi} f(\epsilon, t) \tag{28}$$

where $\nu_{xi} \equiv \nu_x + \nu_i$, ν_x and ν_i are the excitation and ionization collision frequencies, and

$$g(t) = \int_0^\infty (\nu_x + 2\nu_i) f(\epsilon, t)\, d\epsilon \tag{29}$$

The factor of 2 enters in equation (29) since electrons are gained in the distribution following an ionization event in which the ionizing electron is left with energy ϵ and ionizing events in which the ionized electron has an energy ϵ. The approximation for the kinetic equation assumes that the rate at which an electron gains energy in the oscillating electric field can be represented in terms of an "effective" dc field. This approximation has been shown to be reasonable if the radiation frequency is much larger than electron growth rate (Ref. 14).

Defining the Laplace transforms of the distribution function $f(\epsilon, t)$ and the function $g(t)$,

$$\tilde{f}(\epsilon, p) = \int_0^\infty f(\epsilon, t)\, e^{-pt}\, dt \tag{30}$$

$$\tilde{g}(p) = \int_0^\infty g(t)\, e^{-pt}\, dt \tag{31}$$

$$= \int_0^\infty \nu(\epsilon)\, \tilde{f}(\epsilon, p)\, d\epsilon$$

where $\nu(\epsilon) \equiv 2\nu_i(\epsilon) + \nu_x(\epsilon)$. Equation (28) becomes

$$\left(k\, \frac{\partial}{\partial \epsilon} + p + \nu_{xi}\right) \tilde{f}(\epsilon, p) = \delta(\epsilon)\, [\, 1 + \tilde{g}(p)] \tag{32}$$

where the assumption

$$f(\epsilon, o) = \delta(\epsilon) \tag{33}$$

has been used. Dividing equation (32) by $[1 + \tilde{g}(p)]$,

$$\left(k\, \frac{\partial}{\partial \epsilon} + p + \nu_{xi}\right) \tilde{G}(\epsilon, p) = \delta(\epsilon) \tag{34}$$

where

$$\tilde{G}(\epsilon, p) \equiv \frac{\tilde{f}(\epsilon, p)}{1 + \tilde{g}(p)} \tag{35}$$

Multiplying equation (35) by $\nu(\epsilon)$ and integrating over all ϵ.

$$\int_0^\infty \nu(\epsilon)\,\tilde{G}(\epsilon, p)\,d\epsilon = \frac{1}{1 + \tilde{g}(p)} \int_0^\infty \nu(\epsilon)\,\tilde{f}(\epsilon, p)\,d\epsilon \tag{36}$$

Using equation (31), equation (36) can be rewritten as

$$\tilde{g}(p) = \frac{\tilde{N}(p)}{1 - \tilde{N}(p)} \tag{37}$$

where

$$\tilde{N}(p) = \int_0^\infty \nu(\epsilon)\,\tilde{G}(\epsilon, p)\,d\epsilon \tag{38}$$

Taking the Laplace inversion of equation (37),

$$g(t) = \frac{1}{2\pi i} \int_{\beta - i\infty}^{\beta + i\infty} \frac{\tilde{N}(p)}{1 - \tilde{N}(p)}\, e^{pt}\, dp \tag{39}$$

Equation (39) can be integrated by closing the contour in the left half of the complex p-plane giving

$$g(t) = - \frac{\tilde{N}(\gamma)}{\left(\frac{d\tilde{N}}{dp}\right)_{p = \gamma}}\, e^{\gamma t} \tag{40}$$

where γ is the value of p for which $\tilde{N}(p = \gamma) = 1$. Thus, γ is the growth rate for electron production and is determined by the condition $\tilde{N}(\gamma) = 1$.

Solving equation (34) for $\tilde{G}(\epsilon, \gamma)$,

$$\tilde{G}(\epsilon, \gamma) = \frac{1}{k} e^{-\frac{1}{k}\int_0^{\epsilon} (\gamma + \nu_{xi})\, d\epsilon'} \tag{41}$$

Substituting equation (41) into equation (38) and using the relation $\tilde{N}(\gamma) = 1$,

$$\frac{1}{k}\int_0^{\infty} \nu(\epsilon) \exp\left[-\frac{1}{k}\left(\gamma\epsilon + \int_0^{\epsilon} \nu_{xi}(\epsilon')d\epsilon'\right)\right] d\epsilon = 1 \tag{42}$$

Equation (42) can be written in the equivalent form

$$\int_0^{\infty} \left(\frac{\nu_{xi} + \gamma}{k}\right) \exp\left[-\int_0^{\epsilon} \left(\frac{\nu_{xi} + \gamma}{k}\right)\right] d\epsilon'\, d\epsilon$$

$$+ \frac{1}{k}\int_0^{\infty} \nu_i(\epsilon) \exp\left[-\frac{1}{k}\left(\gamma\epsilon + \int_0^{\epsilon} \nu_{xi}(\epsilon')d\epsilon'\right)\right] d\epsilon$$

$$- \frac{\gamma}{k}\int_0^{\infty} \exp\left[-\frac{1}{k}\left(\gamma\epsilon + \int_0^{\epsilon} \nu_{xi}(\epsilon')\, d\epsilon'\right)\right] d\epsilon = 1 \tag{43}$$

The first integral of equation (43) integrates to unity, hence,

$$\gamma = \frac{\int_0^{\infty} \nu_i(\epsilon)\, e^{-\frac{\gamma\epsilon}{k} - \frac{1}{k}\int \nu_{xi}(\epsilon')d\epsilon'}\, d\epsilon}{\int_0^{\infty} e^{-\frac{\gamma\epsilon}{k} - \frac{1}{k}\int_0^{\epsilon} \nu_{xi}(\epsilon')d\epsilon'}\, d\epsilon} \tag{44}$$

When the energy is less than the ionization energy, the numerator of equation (44) is zero, therefore equation (44) can be rewritten as

$$\gamma = \frac{e^{-\bar{\nu}_x \Delta\epsilon/k} \int_{\epsilon_i}^{\infty} \nu_i(\epsilon)\, e^{-\frac{\gamma\epsilon}{k} - \frac{1}{k}\int_{\epsilon_i}^{\epsilon} \nu_{xi}(\epsilon')\, d\epsilon'}\, d\epsilon}{\int_0^{\infty} e^{-\frac{\gamma\epsilon}{k} - \frac{1}{k}\int_0^{\epsilon} \nu_{xi}(\epsilon')\, d\epsilon'}\, d\epsilon} \tag{45}$$

where

$$\bar{\nu}_x \Delta\epsilon = \int_{\epsilon_x}^{\epsilon_i} \nu_x(\epsilon')\, d\epsilon' \tag{46}$$

and ϵ_i, ϵ_x are the ionization and excitation energies, respectively.

Equation (45) can be written in terms of a mean value of the ionization frequency,

$$\gamma = \langle \nu_i \rangle\, e^{-\bar{\nu}_x \Delta\epsilon/k} \left[1 - \frac{B}{1+B}\right] \tag{47}$$

where B is a correction factor describing the deviation of the ionization frequency from its mean value. The value of B can range from B = 0, corresponding to no excitation levels, to B = ∞, corresponding to an infinite excitation frequency. In the case B = 0, the ionization frequency will be just the inverse of the time required for the electron to gain an amount of energy equal to the ionization energy.

$$\langle \nu_i \rangle \approx \frac{k}{\epsilon_i} \tag{48}$$

hence,

$$\gamma = \frac{k}{\epsilon_i} e^{-\bar{\nu}_x \Delta\epsilon/k} \left(1 - \frac{B}{1+B}\right) \tag{49}$$

As a first approximation it will be assumed that B is small and that the growth rate is adequately described by

$$\gamma = \frac{k}{\epsilon_i} e^{-\bar{\nu}_x \Delta\epsilon/k} \tag{50}$$

The electrons in the laser field will gain energy at some distribution varying about the mean rate of energy gain. A general distribution function of the form

$$j(k)\,dk = \left(\frac{\eta}{k_o}\right)^{\eta} \frac{1}{\Gamma(\eta)} k^{\eta-1} e^{-\eta k/k_o}\,dk \tag{51}$$

where η is a numerical factor and k_o is the mean rate of energy gain will be considered. It should be noted that equation (51) satisfies the conditions

$$\int_o^\infty j(k)\,dk = 1$$
$$\int_o^\infty kj(k)\,dk = k_o \tag{52}$$

If a value $\eta = \frac{3}{2}$ is used equation (51) has a Maxwellian shape about the mean energy gain.

The mean growth rate is

$$\bar{\gamma} = \frac{\eta^{\eta} k_o}{\Gamma(\eta)\epsilon_i} \int_0^{\infty} y^{\eta} \, e^{-\left(\eta y + \frac{\bar{\nu}_x \Delta\epsilon}{k_o y}\right)} \, dy \tag{53}$$

where

$$y \equiv k/k_o \tag{54}$$

Making the transformation

$$y = \left(\frac{\bar{\nu}_x \Delta\epsilon}{\eta k_o}\right)^{\frac{1}{2}} e^{\zeta} \tag{55}$$

in equation (53) gives

$$\bar{\gamma} = \frac{\eta^{\eta} k_o}{\Gamma(\eta)\epsilon_i} \left(\frac{\bar{\nu}_x \Delta\epsilon}{\eta k_o}\right)^{(\eta+1)/2} \int_{-\infty}^{\infty} e^{(\eta+1)\zeta} \, e^{-2\left(\frac{\eta \bar{\nu}_x \Delta\epsilon}{k_o}\right)^{\frac{1}{2}} \cosh\zeta} \, d\zeta$$

or

$$\bar{\gamma} = 2 \frac{\eta^{\eta} k_o}{\Gamma(\eta)\epsilon_i} \left(\frac{\bar{\nu}_x \Delta\epsilon}{\eta k_o}\right)^{(\eta+1)/2} \int_0^{\infty} \cosh[(\eta+1)\zeta] \, e^{-2\left(\frac{\eta \bar{\nu}_x \Delta\epsilon}{k_o}\right)^{\frac{1}{2}} \cosh\zeta} \, d\zeta \tag{56}$$

The integral in equation (56) is a well known integral representation for a modified Bessel function of the second kind (Ref. 15)

$$\bar{\gamma} = 2 \frac{\eta^{\eta} k_o}{\Gamma(\eta)\epsilon_i} \left(\frac{z}{2\eta}\right)^{\eta+1} K_{\eta+1}(z) \tag{57}$$

where

$$z = 2\left(\frac{\eta\,\bar{\nu}_x \Delta\epsilon}{k_o}\right)^{\frac{1}{2}} \tag{58}$$

Thus, one can utilize equation (57) to study the average electron growth rate for a very generalized distribution in the rate of energy gain.

For the present study a Maxwellian type of distribution function will be assumed, thus,

$$\bar{\gamma} = \frac{4}{\pi}\left(\frac{\bar{\nu}_x \Delta\epsilon}{\epsilon_i}\right)[z\,k_2(z)] \tag{59}$$

where

$$z\,k_2(z) = \sqrt{\frac{\pi z}{2}}\,K_{5/2}(z) = \frac{\pi}{2}\,e^{-z}\,(1 + 3z^{-1} + 3z^{-2}) \tag{60}$$

and

$$z = \left(\frac{6\,\bar{\nu}_x \Delta\epsilon}{k_o}\right)^{\frac{1}{2}} \tag{61}$$

It should be noted that when z is large enough so that $3z^{-1}$ and $3z^{-2}$ can be neglected in comparison to unity then equation (59) has the particularly simple form

$$\bar{\gamma} \approx 2\left(\frac{\bar{\nu}_x \Delta\epsilon}{\epsilon_i}\right) e^{-z} \tag{62}$$

The only parameter in equations (59) and (62) which involves the laser properties is z, hence, the relation of the threshold field to the state of the propagating medium can be seen quite clearly.

The rate at which an electron gains energy in the field of a laser beam is given by the relation

$$k_o = \frac{e^2E^2}{2m\omega^2}\ \nu_c = 3 \times 10^{-17}\lambda^2\nu_c P\ (\text{watts/cm}^2)\ \frac{\text{ergs}}{\text{sec.}} \tag{63}$$

where λ is the radiation wavelength and P, as indicated, is in units of watts/cm^2. Thus, in the definition of z, there is a ratio of excitation frequency to total collision frequency. In a first approximation it will be assumed that

$$\frac{\bar{\nu}_x}{\nu_c} = \frac{\bar{\sigma}_x}{\sigma_c} \tag{64}$$

and, in analogy to equation (46),

$$\bar{\sigma}_x \Delta\epsilon = \int_{\epsilon_x}^{\epsilon_i} \sigma_x(\epsilon)\, d\epsilon \tag{65}$$

The electronic excitation processes to be considered are shown in Table I along with other relevant information. The excitation of the N_2

Species	State	Ex. Energy(ev)	Ion. Energy(ev)
N_2	$A^3\Sigma_u^+$	6.7	15.8
	$a^1\Pi_g$	8.4	
	$C^3\Pi_u$	11.2	
O_2	$B^3\Sigma_u^-$	6.1	12.2

Table I

Electronic Excitation Processes

vibrational levels is assumed to have a constant cross section of 3×10^{-16} cm^2, over the energy interval 1.9 to 2.8 ev.

For the singlet transitions, the cross section is given by equation (14). Integrating over the energy interval,

$$\bar{\sigma}_x \Delta\epsilon = \frac{\pi e^4}{4\epsilon_x} \int_{\epsilon_x}^{\epsilon_i} \left(\frac{\epsilon - \epsilon_x}{\epsilon^2} \right) d\epsilon$$

$$= \frac{\pi e^4}{4\epsilon_x} \left[\ln \left(\frac{\epsilon_i}{\epsilon_x} \right) - \left(\frac{\epsilon_i - \epsilon_x}{\epsilon_i} \right) \right] \qquad (66)$$

(singlet transition)

The integral for the triplet transitions, using the cross section given by equation (19),

$$\bar{\sigma}_x \Delta\epsilon = 0.461 \frac{\pi e^4}{\epsilon_x^3} \int_{\epsilon_x}^{\epsilon_i} (\epsilon - \epsilon_x) e^{-\epsilon/\epsilon_x} d\epsilon$$

$$= 0.17 \frac{\pi e^4}{\epsilon_x} \left(1 - 2.72 \frac{\epsilon_i}{\epsilon_x} e^{-\epsilon_i/\epsilon_x} \right) \qquad (67)$$

(triplet transitions)

The contributions from each of the levels is given in Table II.

Species	State	$\bar{\sigma}_x \Delta G$ (cm-ev)
N_2	$A^3\Sigma_u^+$	6.38×10^{-16}
	$C^3\Pi_u$	5.89×10^{-17}
	$a^1\Pi_g$	3.12×10^{-16}
	VIB.	2.70×10^{-16}
O_2	$B^3\Sigma_u^-$	3.42×10^{-16}

Table II

Contributions From Each Level

Assuming a composition of 78% N_2 and 21% O_2, the total integral of the excitation cross section over the energy range from threshold to ionization is,

$$\bar{\sigma}_x \Delta\epsilon = 1.07 \times 10^{-15} \text{ cm}^2 - \text{ev} \tag{68}$$

The contribution from the $N_2(C^3\Pi_u)$ and $O_2(B^3\Sigma_u^-)$ (Schumann-Runge band) bands is negligible in comparison to the other states.

The calculated results for the complete breakdown of air, assuming an initial electron density of one electron per cm^3, are shown in figure 9. The results of calculations by Canavan et al. (Ref. 16) are shown for purposes of comparison. The agreement is relatively good but is somewhat higher than the quoted experimental value of $\rho \sim 2 \times 10^9$ watts/cm^2.

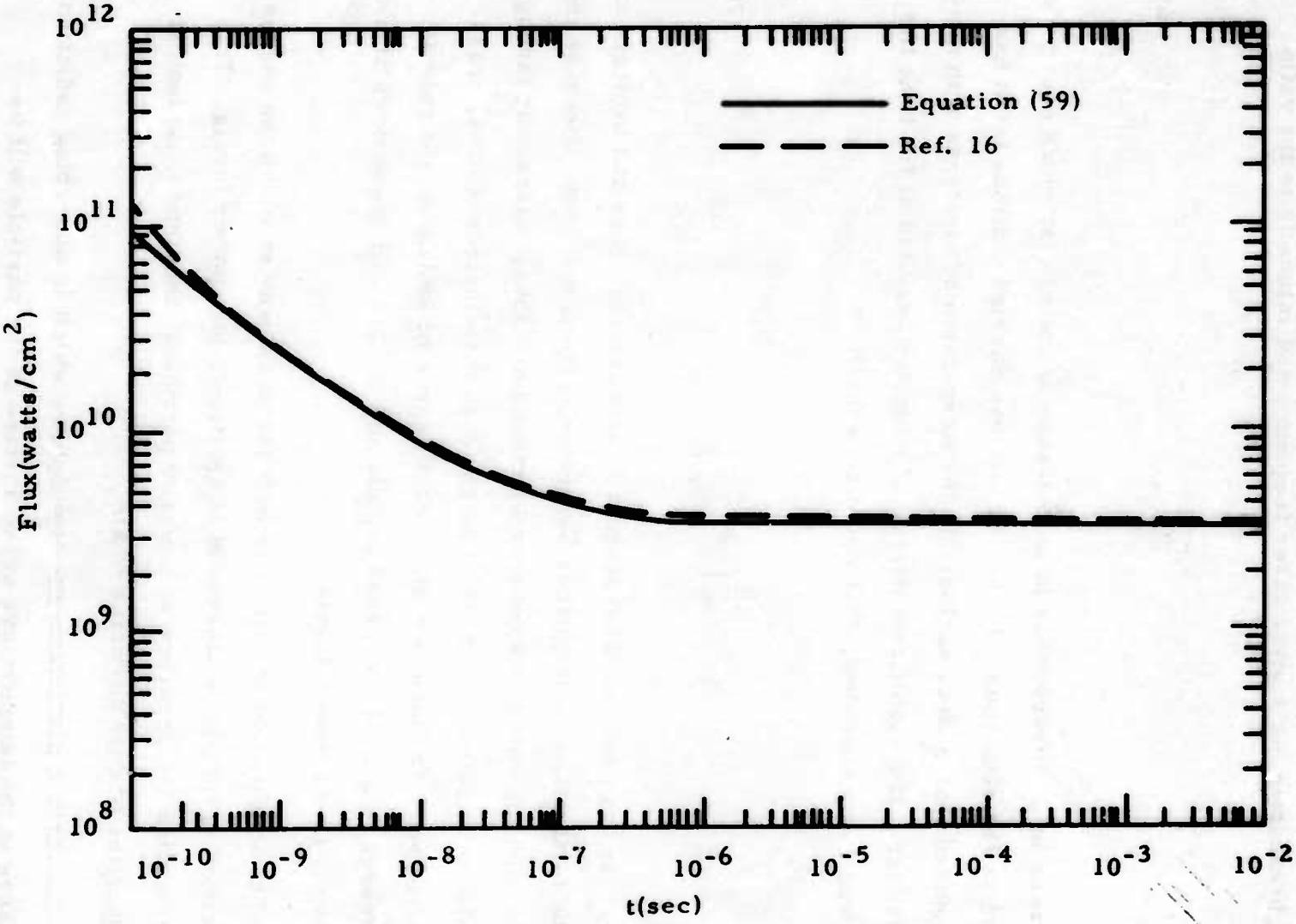


Figure 9. Comparison of Air Breakdown Calculations

Nevertheless, it is clear that the critical parameter in the determination of breakdown for a given laser frequency and intensity is the ratio

$$\frac{\bar{\nu}_x \Delta\epsilon}{\nu_c} \tag{69}$$

An increase in ν_c corresponds to an increase in the rate at which energy is absorbed by the electrons. In the calculations carried out thus far it has been assumed that ν_c was exclusively the interaction of electrons with neutral particles. If a significant degree of ionization existed at the time the energy was being absorbed, this would be written

$$\nu_c = \nu_{en}\left(1 + \frac{\nu_{ei}}{\nu_{en}}\right) \tag{70}$$

where ν_{en} is the total collision frequency between electrons and neutrals and ν_{ei} is the total collision frequency between electrons and ions. Because the cross section for collisions between electrons and ions, interacting through the Coulomb potential, is so much larger than the electron-neutral cross section, as soon as there is a significant degree of ionization the rate at which energy is gained increases rapidly and the air will break down at proportionately lower power levels.

The implication of this is to look for some way in which the degree of ionization in the air is increased at relatively low power levels. This usually implies the presence of impurity particles, thus one must look at the properties of dust particles in air.

Consider a particle in the atmosphere which is absorbing radiation. The change in the temperature at the surface of the particle will be

$$\Delta T \sim \frac{K\varphi}{c_v \rho}\,\Delta t \tag{71}$$

where K is the absorption coefficient, φ is the radiation flux, c_v is the specific heat capacity of the absorbing particle, and ρ is the particle density. As a typical example, one can consider quartz (SiO_2) which has an absorption coefficient of $K \sim 10^4 cm^{-1}$ at $\lambda \approx 10.6\mu$, density $\rho \approx 2.7$ gm/cm^3, and specific heat capacity $c_v \sim 1.3 \times 10^7$ ergs/gm °K. The vaporization temperature for SiO_2 is $T = 2.5 \times 10^3$ °K, thus, in order to raise the temperature of the surface of the particle to the vaporization level in a time $t \sim 10^{-8}$ sec. requires a flux $\varphi \approx 9 \times 10^7$ watts/cm^2. The equilibrium electron density for SiO_2, which exists as SiO in the vapor state, can be obtained from the Saha equation,

$$\frac{N_e^2}{N_n} = 2\left(\frac{2\pi mkT}{k^2}\right)^{3/2} \frac{g_+}{g_a} e^{-\epsilon_i/kT} \qquad (72)$$

where h is the Planck constant and g is the degeneracy. Solving for N_e,

$$N_e = \frac{S(T)}{2}\left[-1 + \sqrt{1 + \frac{4N_o}{S(T)}}\right] \qquad (73)$$

where S(T) is just the right hand side of equation (72). Taking a particle density of $N_o \approx 4 \times 10^{22} cm^{-3}$ and an ionization energy of $\epsilon_i \approx 10.8$ ev, the equilibrium electron density at vaporization temperature will be $N_e \sim 10^{11} cm^{-3}$. While this is a significant increase in electron density, it is probably not enough to lead to a major change in the breakdown power levels. Thus, one must do a somewhat more careful study of the heating and subsequent vaporization of dust particles with particular attention to various mechanisms for electron enhancement.

SECTION III

THE HYDRODYNAMIC EQUATIONS

Two separate models, one utilizing a single temperature for both ions and electrons and the other model treating the temperatures separately, have been developed for the description of dirty air breakdown. For this report, however, only the more realistic two-temperature model has been used for calculations. In both models a symmetry is assumed so that the problem can be treated in one dimension. The Lagrangean form of the one-dimensional hydrodynamic equations is used. The first three equations are essentially the same in both models (Ref. 17),

$$\frac{\partial R}{dt} = U \tag{74}$$

$$\frac{\partial U}{dt} = - R^{\alpha - 1} \frac{\partial}{\partial M} (P + Q) \tag{75}$$

$$V = \frac{1}{\rho} = \frac{1}{\alpha} \frac{\partial R^{\alpha}}{\partial M} \tag{76}$$

expressing the definition of velocity U, the conservation of momentum and the conservation of mass. The artificial viscosity Q is necessary for numerical stability of the difference equations discussed in Appendix I. The geometry of the single dimension is plane, cylindrical or spherical for α = 1, 2 or 3, respectively, and M is the Lagrangean mass coordinate.

The one-temperature model is based on the assumption that the free electrons are in thermal equilibrium with the neutral molecules and ions of the shock heated air and blow off vapor. The electron density is determined by the Saha equation. The energy equation of hydrodynamics is

$$\frac{\partial E}{\partial t} = -\bar{P}\frac{\partial V}{\partial t} + \varphi\bar{K} + \frac{\partial}{\partial M}\left[\chi R^{\alpha-1}\frac{\partial T}{\partial R}\right] \qquad (77)$$

where χ is the thermal conductivity, φ is the laser flux and $\bar{K}$ is the effective mass absorption coefficient for the material and free electrons. The hydrodynamic description is completed by air equation of state giving the pressure P as a function of the density ρ and specific internal energy E.

In the two-temperature model, it is assumed that the electrons are in a Boltzmann distribution at temperature T_e. The electrons interact with the neutral molecules and ions which are in a second Boltzmann distribution with temperature T_i. The total pressure in the momentum equation is the sum of the partial pressures of the electrons and ions.

$$P = P_e + P_i \quad . \qquad (78)$$

For notation, the neutral molecules and ions are referred to collectively as "ions."

The energy equation now has separate components for the ions and electrons,

$$\frac{\partial E_i}{\partial t} = -\bar{P}_i\frac{\partial V}{\partial t} + \frac{\partial}{\partial M}\left[\chi_i R^{\alpha-1}\frac{\partial T_i}{\partial R}\right] + \varphi\bar{K}_i + \bar{\nu}_c 2\left(\frac{m_e}{m_i}\right)\left(E_e - E_i\right) \qquad (79)$$

$$\frac{\partial E_e}{\partial t} = -\bar{P}_e\frac{\partial V}{\partial t} + \frac{\partial}{\partial M}\left[\chi_e R^{\alpha-1}\frac{\partial T_e}{\partial R}\right] + \varphi\bar{K}_e - \bar{\nu}_c 2\left(\frac{m_e}{m_i}\right)\left(E_e - E_i\right) \qquad (80)$$

$$- \frac{\bar{\nu}_x N_e \epsilon_x}{\rho} - \frac{\bar{\nu}_i N_e \epsilon_i}{\rho} - \bar{\nu}_a E_e$$

where $\bar{P}_i + \bar{P}_e = P + Q$, $\bar{\nu}$ represents a collision frequency, ϵ is an energy, and E is a specific energy. The subscripts c, x, a and i represent elastic

collisions, excitation collisions, attachment, and ionization, respectively.

The change in the electron density is given by

$$\frac{\partial N_e}{\partial t} = \rho \frac{\partial}{\partial M}\left[D_e R^{\alpha=1} \frac{\partial N_e}{\partial R}\right] + \bar{\nu}_i N_e - \bar{\nu}_\alpha N_e \qquad (81)$$

where D_e is the electron diffusion coefficient. Functional forms for these parameters are discussed in the next section.

1. COLLISION FREQUENCIES

The energy losses within the electron and ion gases are expressed in terms of the various collision frequencies averaged over some suitable distribution of energies. For the calculations to be carried out in this report, it will be assumed that the components of the gas have a Maxwellian velocity distribution about some temperature representative of the component under consideration, thus

$$\bar{\nu} = N \frac{\left(\frac{2}{\pi m}\right)^{\frac{1}{2}}}{(kT)^{3/2}} \int \epsilon\, \sigma(\epsilon)\, e^{-\epsilon/kT}\, d\epsilon \qquad (82)$$

where N is the density of target particles, m is the mass of the colliding particle (in some cases the reduced mass), and σ is the cross section for the interaction under consideration. The integration is carried out over the appropriate energy range.

a. Electronic Excitation

Utilizing equation (14) of Section II for the cross section for singlet transitions,

$$\bar{\nu}_x = \frac{N_m e^4}{\epsilon_x (kT_e)^{3/2}} \left(\frac{\pi}{2m}\right)^{\frac{1}{2}} \int_{\epsilon_x}^{\infty} \left(\frac{\epsilon - \epsilon_x}{\epsilon}\right) e^{-\epsilon/kT}\, d\epsilon \qquad (83)$$

where m is the electron mass, e is the electron charge, and T_e is the electron temperature. The integration is straightforward,

$$\bar{\nu}_x = \frac{N_m e^4}{\epsilon_x}\left(\frac{\pi}{2mkT_e}\right)^{\frac{1}{2}} e^{-\epsilon_x/kT_e}\left[1 - \frac{\epsilon_x}{kT_e} E_1\left(\frac{\epsilon_x}{kT_e}\right) e^{\epsilon_x/kT_e}\right] \qquad (84)$$

(singlet transition)

where $E_1(z)$ is an exponential integral of the first kind. In the special case $\epsilon_x >> kT_e$,

$$\bar{\nu}_x = \frac{N_m e^4}{2}\left(\frac{\pi kT_e}{2m}\right)^{\frac{1}{2}} e^{-\epsilon_x/kT_e} \qquad (85)$$

$$\epsilon_x^2$$

(singlet transition)

A similar calculation for the triplet transitions gives

$$\bar{\nu}_x = \frac{0.340\, e^4 N_m}{(kT_e + \epsilon_x)}\left(\frac{\pi}{2mkT_e}\right)^{\frac{1}{2}} e^{-\epsilon_x/kT}\left[1 + \frac{\left(1 - \epsilon_x/kT_e\right)\left(2 + \frac{\epsilon_x}{kT_e}\right)}{\left(1 + \epsilon_x/kT_e\right)}\right] \qquad (86)$$

(triplet transition)

b. Vibrational Excitation

The cross section for the resonant excitation of N_2 vibrational levels has been assumed to be constant over the energy range of interest, hence,

$$\bar{\nu}_x = N_m \sigma_v \left(\frac{8kT_e}{\pi m}\right)^{\frac{1}{2}}\left[\left(1 + \frac{\epsilon_1}{kT_e}\right) e^{-\epsilon_1/kT_e} - \left(1 + \frac{\epsilon_2}{kT_e}\right) e^{-\epsilon_2/kT_e}\right] \qquad (87)$$

where $\sigma_v \approx 3 \times 10^{-16} cm^2$., $\epsilon_1 \approx 1.8$ ev and $\epsilon_2 \approx 2.8$ ev.

c. Ionization Frequencies

For N_2, using equation (25) for the cross section,

$$\bar{\nu}_i(N_2) = \frac{1.64 \times 10^{-15} \epsilon_i^2}{\left(kT_e\right)^{3/2}} \left(\frac{2}{\pi m}\right)^{\frac{1}{2}} \frac{e^{-\epsilon_i/kT_e}}{\left(1 + 5\frac{\epsilon_i}{kT_e}\right)} N_M \times \left[1 + \frac{54\left(1 - \frac{5}{9}\frac{\epsilon_i}{kT_e}\right)\left(1 + \frac{5}{6}\frac{\epsilon_i}{kT_e}\right)}{\left(1 + \frac{5\epsilon_i}{kT_e}\right)^2}\right] \tag{88}$$

and for O_2, using equation (26),

$$\bar{\nu}_i(O_2) = \frac{1.74 \times 10^{-15} \epsilon_i^2}{\left(kT_e\right)^{3/2}} \left(\frac{2}{\pi m}\right)^{\frac{1}{2}} \frac{e^{-\epsilon_i/kT_e}}{\left(1 + 15\frac{\epsilon_i}{kT_e}\right)} N_m \times \left[1 + \frac{464\left(1 - \frac{15}{29}\frac{\epsilon_i}{kT_e}\right)\left(1 + \frac{15}{16}\frac{\epsilon_i}{kT_e}\right)}{\left(1 + \frac{15\epsilon_i}{kT_e}\right)^2}\right] \tag{89}$$

d. Electron-Ion Elastic Collision Frequency

The elastic collision frequency for two particles interacting through a Coulomb potential can be evaluated from classical considerations (Ref. 18). The result, after averaging over the relative velocities is,

$$\bar{\nu} = \frac{4\pi N e^4}{m_o^{1/2}(2kT)^{3/2}} \ln \Lambda \tag{90}$$

where m_o is the reduced mass and

$$\Lambda \equiv \frac{3kT}{e^2} r_D \tag{91}$$

$$r_D \equiv \left[\frac{kT}{4\pi e^2 (N_i + N_e)} \right]^{\frac{1}{2}} \tag{92}$$

The length, r_D, is the well known Debye length and N_i, N_e are the ion and electron number densities, respectively. For the case of electron-ion collisions, the reduced mass in equation (90) is replaced by the electron mass.

2. AMBIPOLAR DIFFUSION COEFFICIENT

When the Debye length in a plasma is small in comparison to the characteristic diffusion length, the free electron diffusion coefficient must be replaced by the ambipolar diffusion coefficient. The Debye length is defined by

$$r_D = \left(\frac{kT}{8\pi N_e e^2} \right)^{\frac{1}{2}} \tag{93}$$

For typical conditions of interest, $T \sim 3 \times 10^3$ °K, $N_e \geq 10^{15} cm^{-3}$, the Debye length is $r_D \sim 10^{-5}$cm. The free electron diffusion coefficient under the same conditions will be $D_e \sim 10^2 cm^2/sec$. The characteristic relaxation time of interest will be $\tau \sim 10^{-8}$ sec., hence,

$$L_D \sim \sqrt{D_e \tau} \sim 10^{-3} \text{cm.} \tag{94}$$

Thus, one must consider the space charge effects on the diffusion of the electrons.

To do so one notes that the effect of the space charge will be to cause the electron to pull an ion along with it. This can be viewed as increasing the mass of the electron in the electron diffusion coefficient by the mass of the ion. The ion collision cross section with heavy particles will be about twice that of the electron cross section, hence,

$$\frac{\left(\frac{8kT_e}{\pi M_i}\right)^{\frac{1}{2}}}{N_a \sigma_i} = 2D_i \tag{95}$$

This is correct only when the ion and electron temperatures are equal. However, for purposes of the present report the temperatures will be close enough over times of interest so that equation (95) can be used.

3. FREE-FREE ABSORPTION COEFFICIENT

The interaction between a current and an electric field is described by the Maxwell's equations,

$$c\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{96}$$

$$\nabla \times \vec{B} = \frac{4\pi \vec{j}}{c} + \frac{1}{c}\frac{\partial \vec{E}}{\partial t} \tag{97}$$

Reducing equations (96) and (97) to a single wave equation

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial \vec{j}}{\partial t} \tag{98}$$

where $\vec{j}$ is the current density,

$$\vec{j} = - Ne\vec{v} \tag{99}$$

where N is the electron density and $\vec{v}$ is the electron velocity.

The motion of an electron is an electric field is described by the equation of motion

$$\frac{d\vec{v}}{dt} + \nu_c \vec{v} = - \frac{e\vec{E}}{m} \tag{100}$$

where ν_c is the collision frequency. Assuming that the vector quantities all have the time behavior $e^{-i\omega t}$ where ω is the oscillation frequency, reduces equations (98) and (100)

$$\vec{v} = - \frac{e\vec{E}(\nu_c + i\omega)}{m(\omega^2 + \nu^2)} \tag{101}$$

$$\nabla^2 \vec{E} + \frac{\omega^2}{c^2} \vec{E} = i \frac{4\pi}{c^2} \omega N_e \vec{v} \tag{102}$$

Substitution of equation (101) into equation (102)

$$\nabla^2 \vec{E} + k^2 \vec{E} = 0$$

where

$$k = \frac{\omega}{c}\left[1 + i\,\frac{4\pi e^2 N(\nu_c + i\omega)}{m\omega(\omega^2 + \nu^2)}\right]^{\frac{1}{2}} \tag{103}$$

Assuming that the second term in the square brackets is small in comparison to unity, one can use the usual approximation

$$k \approx \frac{\omega}{c}\left[1 + i\,\frac{2\pi e^2 N(\nu_c + i\omega)}{m\omega(\omega^2 + \nu^2)}\right] \tag{104}$$

The absorption coefficient will be just the imaginary portion of equation (104)

$$K_{ff} = \frac{2\pi N_e^2}{mc(\omega^2 + \nu^2)}\,\nu_c \tag{105}$$

This is the usual form for the free-free absorption coefficient. The mass absorption coefficient of equation (105) is just

$$\bar{K}_e = \frac{K_{ff}}{\rho} \tag{106}$$

SECTION IV

PARTICLE DESCRIPTION AND INTERACTIONS

The size of particles in dry atmospheric aersols range from about 0.005μ to 20μ. Smaller particles are quickly attached to larger particles and are found only in a region of constant production. Particles larger than about 20μ are lost by sedimentation and are found only in the vicinity of a source. The density of particles versus radius shows a maximum near 10μ (Ref. 19).

Air breakdown experiments have been performed on particles in the range 40-100μ (Ref. 2). In order to compare our results with these experiments, computer runs were made for particles ranging in diameter from 12μm to 96μm.

1. ENERGY ABSORPTION FROM THE LASER BEAM

Energy is absorbed from the laser beam by two processes: mass absorption by the particle material, and inverse bremsstrahlung absorption by free electrons. As the particles are heated and ionized, the mechanism for absorption of energy from the laser beam shifts from the first to the second kind.

a. Mass Absorption

In general, it is necessary to know the mass absorption coefficients of the various types of dust particles which may be encountered in the atmosphere. Specifically under consideration are the materials Al_2O_3, SiO_3, C. and NaCl as representative of the common types of particles found in the atmosphere. The absorption coefficient is needed as a function of wavelength (at least for specific laser wavelengths of interest), as a function of particle material temperature, and as a function of material density as the particle melts and vaporizes.

Data of the kind needed are scarce. Data are available for normal density solids in the wavelength range of particular interest in optical (Ref. 20) and infrared (Ref. 21) applications at room temperatures. Good data for solids at elevated temperatures are known only for sapphire (Ref. 22) in the 0.5 to 6μm wavelength range. An analysis of the absorption coefficient of quartz vapor (SiO) in air has been given for 10.6μm wavelength as a function of temperature and density (Ref. 23). The reflection coefficient for neodymium laser light on a carbon target and targets of other materials has been reported as a function of laser pulse power (Ref. 24) level. Common salt is quite transparent at most wavelengths of interest. In all, the available empirical data are far from complete, and further experimental measurements in this area would be highly desirable. The known information is summarized below.

(1) SiO_2

Extensive data exist (Refs. 20 and 25) on the values of the absorption coefficient of SiO_2 in its various forms at room temperature for wavelengths in the range from 0.2 to 4.8μm. The temperature dependence is not known to have been measured, however, as it has for sapphire. When SiO_2 is heated in air, it forms SiO vapor as a result of oxidation. The extinction coefficient of SiO has been measured (Ref. 26) as a function of wavelength from 0.24 to 14μm. A plot of the absorption coefficient from these data is shown in figure 11 for SiO. Also shown in the figure is the room temperature absorption coefficient for silica, and one point measured at AFWL for fused silica at 2.8μm. A value of the absorption coefficient between 2000° and about 4000°K for normal density ($\rho = 2.13$ gm/cc) SiO derived from the work of Chang, Drummond, and Hall is also shown for comparison. It falls below that for the room temperature material despite being for elevated temperatures. Both values were used in the computer runs made.

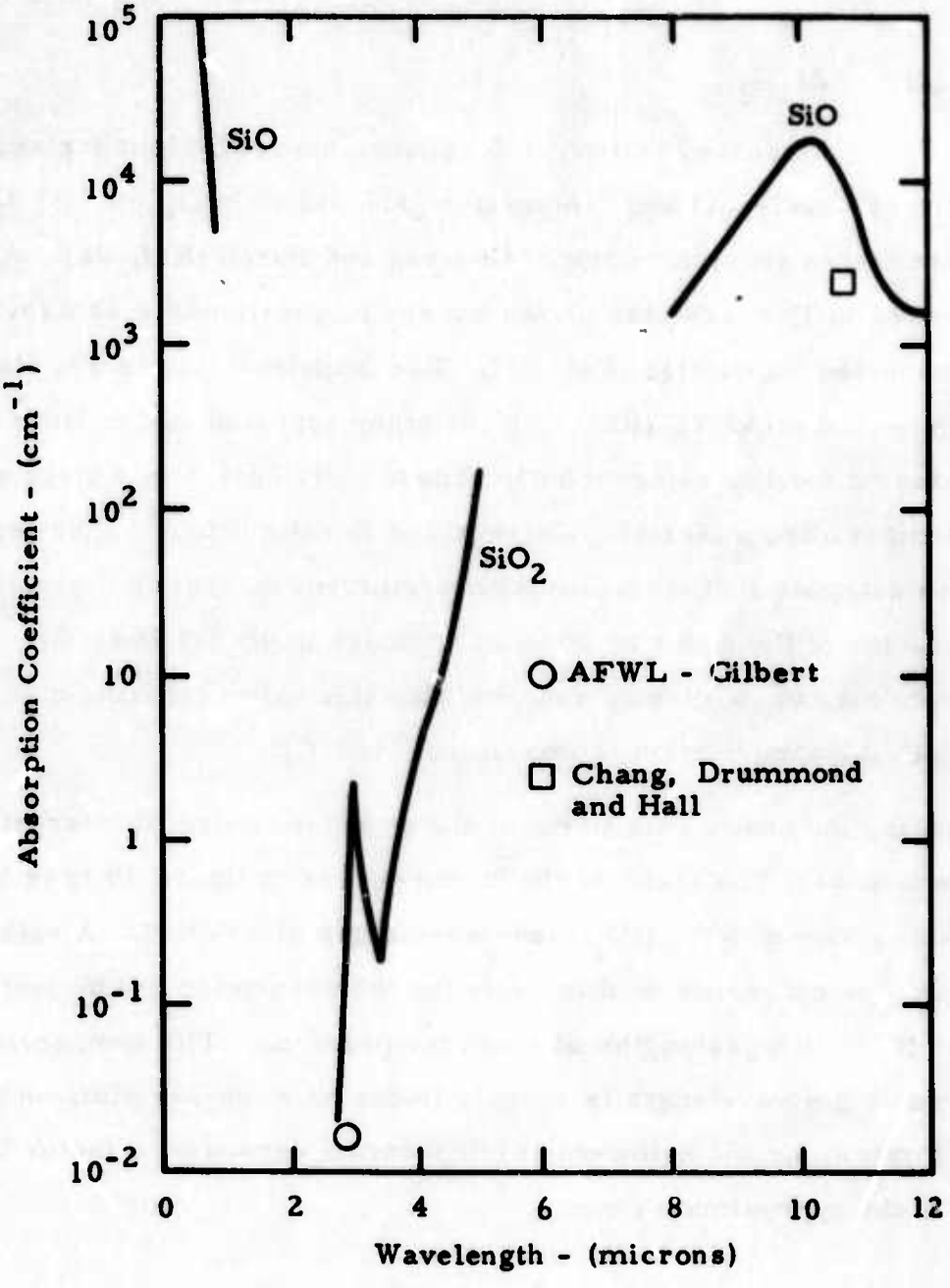


Figure 11. Absorption Coefficient for SiO_2 and SiO as a Function of Wavelength

The density dependence of the absorption coefficient employed was of the form $(\rho/\rho_o)^n$, where n was chosen to equal two for SiO in conformance with the value given for diatomic molecules by Chang, Drummond, and Hall.

(2) Al_2O_3

Measured values of the absorption coefficient for sapphire as a function of wavelength and temperature are shown in figure 10. Most of these data comes from the work of Gryvnak and Burch (Ref. 22). A few points reported in 1936 are also shown but are of questionable accuracy because of suspected impurities (Ref. 27). Two additional points are shown. One was measured at AFWL (Ref. 28); the other reported as the limit of the transmission band in reference 21. These individual points were all for room temperature material. Gryvnak and Burch (Ref. 22) also report a qualitative estimate that the absorption coefficient increases discontinuously by a factor of the order of 30 when sapphire melts (at 2045°C). As a first approximation, it may be assumed that this value remains about constant up to the vaporization temperature (2980°C).

Extending the known data to cover the complete range of interest is largely speculative. The trend of the known curves in figure 10 have been extrapolated as shown to the CO_2 laser wavelength of interest. A value of about $1.0 \mu m^{-1}$ is estimated on this basis for the absorption coefficient of sapphire at 10.6 μm wavelengths at room temperature. The temperature dependence at 5 μm wavelength is roughly linear on a log-log plot, so an empirical formula for the below-melt temperature dependence factor is suggested of the approximate form.

$$f(T) \approx \exp\left[\frac{T - 300}{790}\right] \text{ for } T < 2318°K \qquad (107)$$

where T is the temperature in degrees Kelvin and f(T) is a multiplying factor for the room temperature absorption coefficient. Above 2318°K a

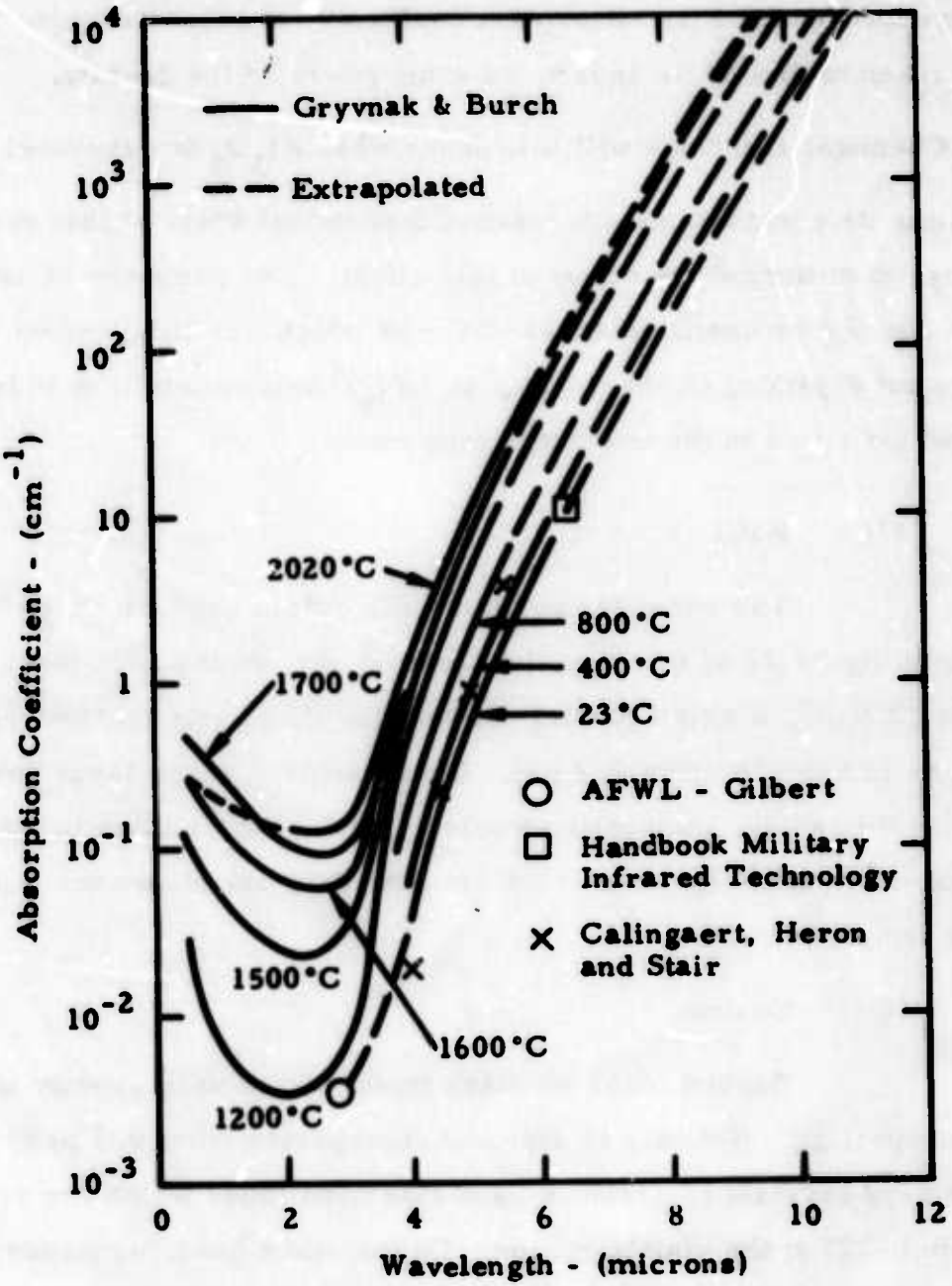


Figure 10. Absorption Coefficient for Al_2O_3 as a Function of Wavelength

constant factor of the order of 400 can be assumed to multiply the room temperature value of the absorption coefficient.

In conformity with the findings of Chang, Drummond and Hall (Ref. 23), the dependence of the absorption coefficient of sapphire vapor can be further taken to depend on density as some power of the density, $\left(\frac{\rho}{\rho_o}\right)^n$. Chemical reactions will take place when Al_2O_3 is vaporized in air which alter its composition. A detailed theoretical study of this problem was considered beyond the scope of this effort. For purposes of estimation, an absorption coefficient was assumed which was independent of temperature but depended on the density as $(\rho/\rho_c)^2$ in conformity with the one employed for silica in the computer runs made.

(3) NaCl

The room temperature absorption coefficient for NaCl is shown in figure 12 at wavelengths above 10 μm. At the CO_2 laser wavelength of 10.6 μm, a small particle of NaCl is effectively transparent, and it remains so down to about 0.2 μm. Consequently, at the laser powers of interest in this study, the heating would be very slow relative to that of the other materials investigated at 10.6 μm. No data are known for NaCl at elevated temperatures.

(4) Carbon

Carbon takes on many forms which vary greatly in their optical properties. Not only is diamond transparent from 0.3 to 80 μm, but very pure crystals of graphite have also been found which are transparent (Ref. 28) in the visible region. On the other hand, usual forms of carbon are opaque. One reason given for this is the density of dislocations in graphite crystals determines the ease with which light can pass through them. Another reason given is that most carbon is very porous, and the light entering it is multiply scattered before it can escape and is thus absorbed. Soot, which may have a density two order of magnitude below

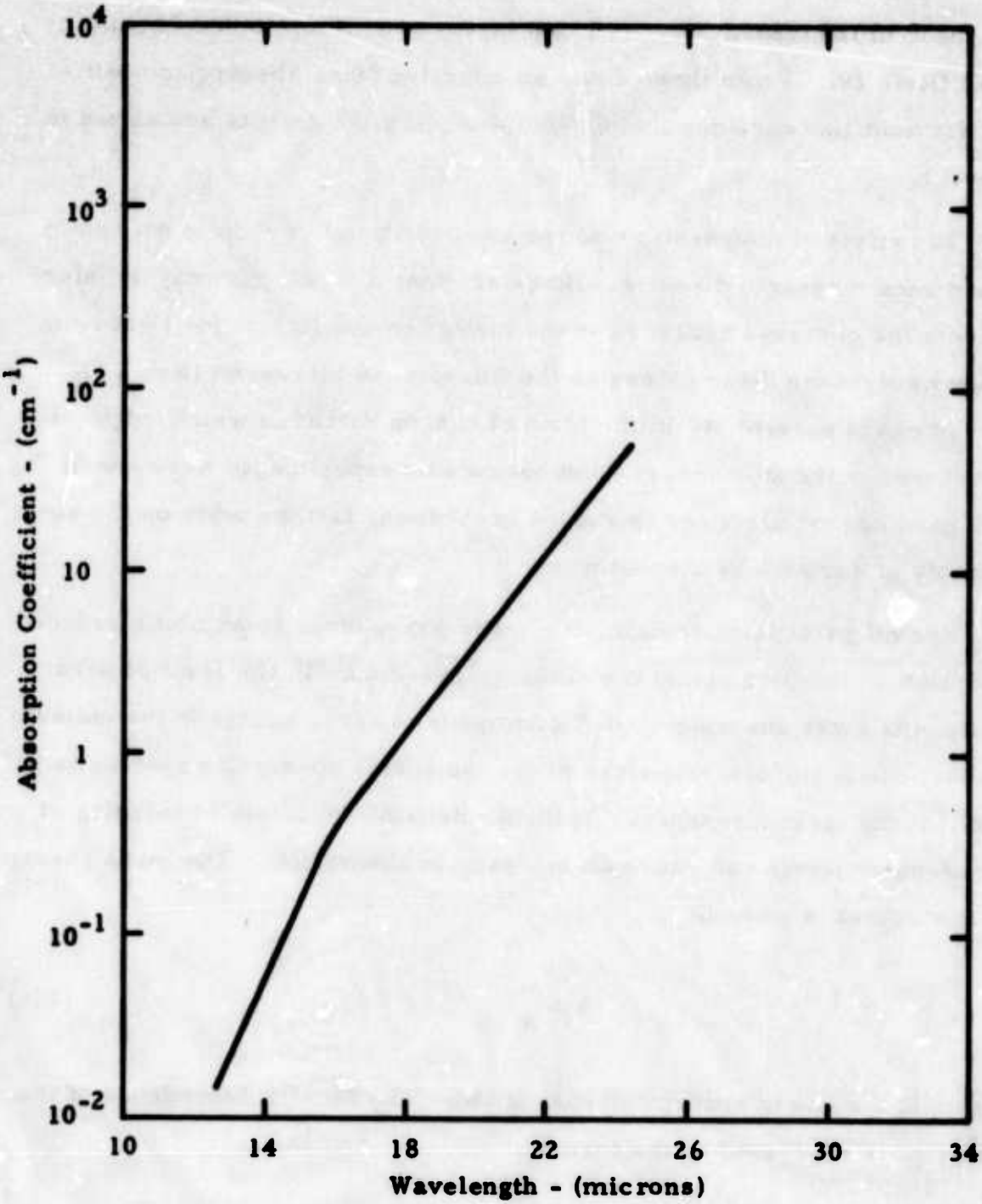


Figure 12. Absorption Coefficient for NaCl as a Function of Wavelength

solid graphite crystals, is particularly porous, and as a consequence is used as an antireflecting coating in the form of lampblack. A study has been made of the transmittance of thin layers of soot and other carbon blacks (Ref. 29). From these data, an effective mass absorption coefficient for soot like carbon may be calculated, and the results are shown in figure 13.

The effect of temperature on the absorptivity of carbon is not known to have been measured directly. However, that it increases may be inferred from the observed decrease in the reflection coefficient for light from intense neodymium laser pulses as the fluence was increased (Ref. 24). Because of the uncertainty in the form of carbon particles which might be encountered in the atmosphere, and because no experiments were known which used carbon particles to reduce breakdown, further work on the absorptivity of carbon was discontinued.

For all particle materials, the vapor phase tends to exhibit a reduced absorption as the band structure of the solid is lost. In the limit of zero density, the mass absorption coefficient goes to zero, except in the unlikely case that one of the discrete lines of the molecular absorption spectra is exactly at the laser frequency. At higher density, collision broadening of the molecular levels can cause an increase in absorption. The mass absorption coefficient is given by

$$A = \frac{a}{\rho} \tag{108}$$

where a is the linear absorption coefficient. The density dependence of the absorption is expressed in the form

$$a(\rho) = a_0 \left(\frac{\rho}{\rho_0}\right)^n \tag{109}$$

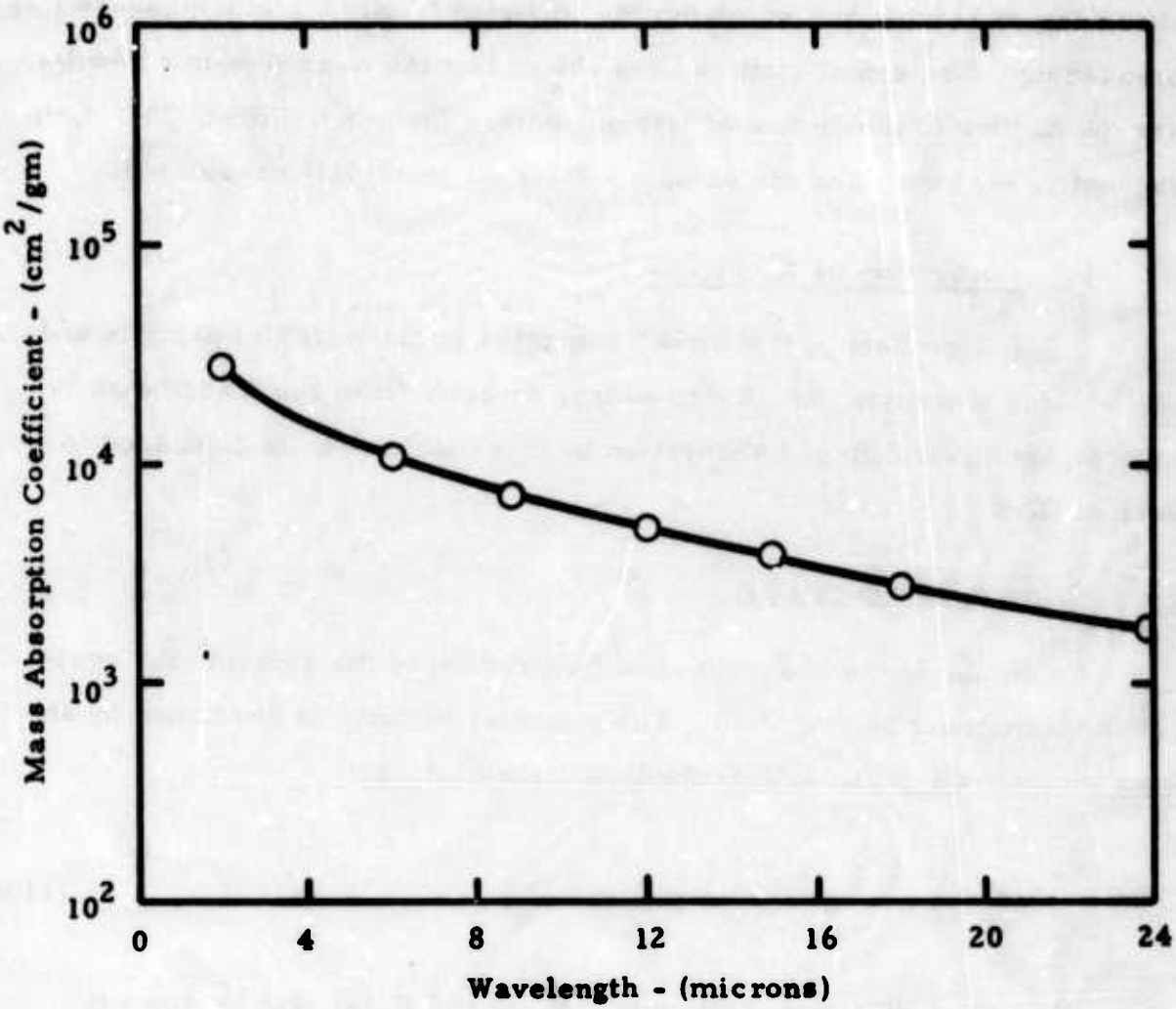


Figure 13. Mass Absorption Coefficient for Carbon (soot) as a Function of Wavelength

where a_0 is the absorptivity of the solid, ρ_0 is the density of the solid, and n is to be determined. For X-ray deposition, the energy is deposited in the atomic electrons, n = 1 and the mass absorption is independent of density. For laser deposition in the vapor from a quartz particle n = 2 (Ref. 23). For this diatomic molecule, one density factor is from the density of absorbing molecules and one from the collision frequency which causes line broadening. Numerical studies have shown that the hydrodynamic results are insensitive to the choice of this parameter for $n \geq 1.5$ (Ref. 30). Consequently, we have used the value n = 2 for all materials considered.

b. Absorption by Electrons

Independent of the mass absorption of the material particle and vapor, free electrons can absorb energy directly from the laser beam by inverse bremsstrahlung. Absorption by this mechanism is discussed in Section III-3.

2. EQUATION OF STATE

To complete the hydrodynamic description of the system, the equation of state must be specified. The material particle is described by an equation of state of the Mie-Grüneisen form (Ref. 31)

$$P = P_0 + \Gamma \rho (E - E_0) \tag{110}$$

where Γ is the Grüneisen coefficient. $P_0(\rho)$ and $E_0(\rho)$ give the density dependence of the pressure and energy density at a reference temperature. In the compressional region,

$$\mu = \frac{\rho}{\rho_o} - 1 > 0 \tag{111}$$

the pressure is given by

$$P = P_H(\mu)\left[1 - \frac{1}{2}\Gamma\mu\right] + \Gamma\rho\left[E - E_o\right] \tag{112}$$

where

$$P_H(\mu) = C\mu + D\mu^2 + S\mu^3 \tag{113}$$

and the Hugoniot coefficients are determined from experimental data (Ref. 32). In the expansion region,

$$\eta = \frac{\rho}{\rho_o} < 1 \tag{114}$$

$$P = \rho\left[\gamma_1 + (\Gamma - \gamma_1)\eta^{\frac{1}{2}}\right]\left\{E - E_S\left[1 - \exp\frac{F}{\eta}(1 - \frac{1}{\eta})\right]\right\} \tag{115}$$

where E_S is the sublimation energy,

$$\gamma_1 = \gamma - 1 \tag{116}$$

and γ is the ratio of specific heats for the vapor. The factor

$$F = \frac{C}{\Gamma E_S \rho_o} \tag{117}$$

ensures continuity with the compressional equation as $\eta \to 1$.

The expansion equation must also allow for a change of phase as the material is heated. In the above expression, the phase transition is included only in an average or smoothed way as discussed in Ref. 32. Near normal density and energy density, the expansion equation gives a pressure appropriate to the solid. At low densities and high-energy density, the expansion pressure approaches that of an ideal gas. For the problem of dirty air breakdown, the region near the phase transition is critical for it controls

the rate of material vaporization, blowoff and subsequent shock heating. There is experimental evidence (Ref. 33) that for some materials the blow-off consists of macromolecular complexes rather than single molecules. This would completely change the above model; changing the effective sublimation energy of the solid and the γ of the vapor.

For the air surrounding the dust particle, two separate models have been used. In the first model, the air is assumed to be in local thermodynamic equilibrium, including the various molecular processes. The pressure, equilibrium electron density and effective specific heat ratio for shocks are computed as functions of density and energy density using an interpolation scheme on tables given by Gilmore (Ref. 34).

The validity of the equilibrium assumption is questionable since the rate constants for the molecular processes are often long compared to laser pulse times of a few microseconds. For example, at standard atmospheric density and 3000°K, the reaction

$$N_2 + O_2 \rightleftarrows 2NO \tag{118}$$

requires about 10^{-4} second to reach equilibrium (Ref. 35). This particular reaction is important for the equilibrium electron density because the ionization potential of NO at 9.25 ev (Ref. 36) is the lowest of the molecular species found in air.

For laser pulses that are short compared to the molecular reaction times, a better approximation to the air equation of state is obtained by assuming no molecular dissociation or reaction. This is the basis of the second model in which the pressure is given by the ideal gas equation. The equilibrium electron density is computed using the Saha equation with a single ionization potential of 12.05 ev for O_2.

Calculations were carried out using both models. Over the times of the order of 10^{-7} sec. there was no significant difference in the answers

obtained from the two models. The results presented in later sections were all obtained from the second model.

The equation of state for the electron gas is just that of an ideal monatomic gas.

$$P_e = (\gamma_e - 1)\, \rho\, E_e \tag{119}$$

where $\gamma_e = \frac{5}{3}$ and E_e is the electron internal energy per gram of material.

Properties of the materials studied are listed in Table III. The parameter values employed in the CELAB computer runs are given.

3. THERMAL CONDUCTIVITY

In general, the thermal conductivity of the dust particle will depend on the temperature. We have assumed the functional form

$$\chi(T) = \chi_o\, T^{\beta} \tag{120}$$

for the temperature dependence. We have fit limited experimental data on quartz and sapphire to determine values of χ_o and β for the two materials. A plot of the fitted conductivity is shown in figure 14.

The thermal conductivity of air shows a general increase from room temperature to about 7000°K, then decreases between 7000 and 10,000°K as shown in figure 15. The data are too complicated to be fit with the functional form of equation (120) with a single value of the parameters over the entire temperature range. We have divided the data into four separate regions and fit each region with the form of equation (120). For any given temperature, a table look-up is used to determine the values of the coefficients and the conductivity computed from the above form.

The thermal conductivity of the free electron gas can be computed from the kinetic theory of gases (Ref. 37).

TABLE III

MATERIAL CONSTANTS

Property	Air	S_iO_2	Al_2O_3
Density (gm/cc)	1.292×10^{-3}	2.65	3.97
γ	1.3802	1.25	1.25
Atomic Weight	28.98	60.06	102.0
Thermal Conductivity Constant (ergs/cm sec $^\circ K^{B+1}$)	31.4	1820	1820
β	0.78	0.74	0.74
Linear Absorption Coeff (cm^{-1})	0.	1×10^4	1×10^4
Grüneisen Coefficient	---	0.78	0.78
Hugoniot Constants			
C ($dyne/cm^2$)	---	8.7×10^{11}	4.65×10^{12}
D ($dyne/cm^2$)	---	0	0
S ($dyne/cm^2$)	---	0	0
Sublimation Energy (erg/gm)	---	2.35×10^{11}	2.35×10^{11}
Specific Heat Solid (erg/gm deg)	---	1.25×10^7	1.72×10^7
Vaporization Temp ($^\circ K$)	---	2503	3253
Work Function (ev)	---	4.58	3.77
Ionization Energy (ev)	12.05	10.8	7.7
Slope of Ionization Cross Section (πa_0^2/ev)	4.5×10^{-2}	8.3×10^{-2}	8.3×10^{-2}
Vibration Threshold (ev)	1.8	1.8	1.8
Vibration Bandwidth (ev)	1.4	1.4	1.4
Vibration Quanta (ev)	0.26	0.26	0.26
Vibrational Cross Section ($1 \times 10^{-15} cm^2$)	26.60	26.60	26.60

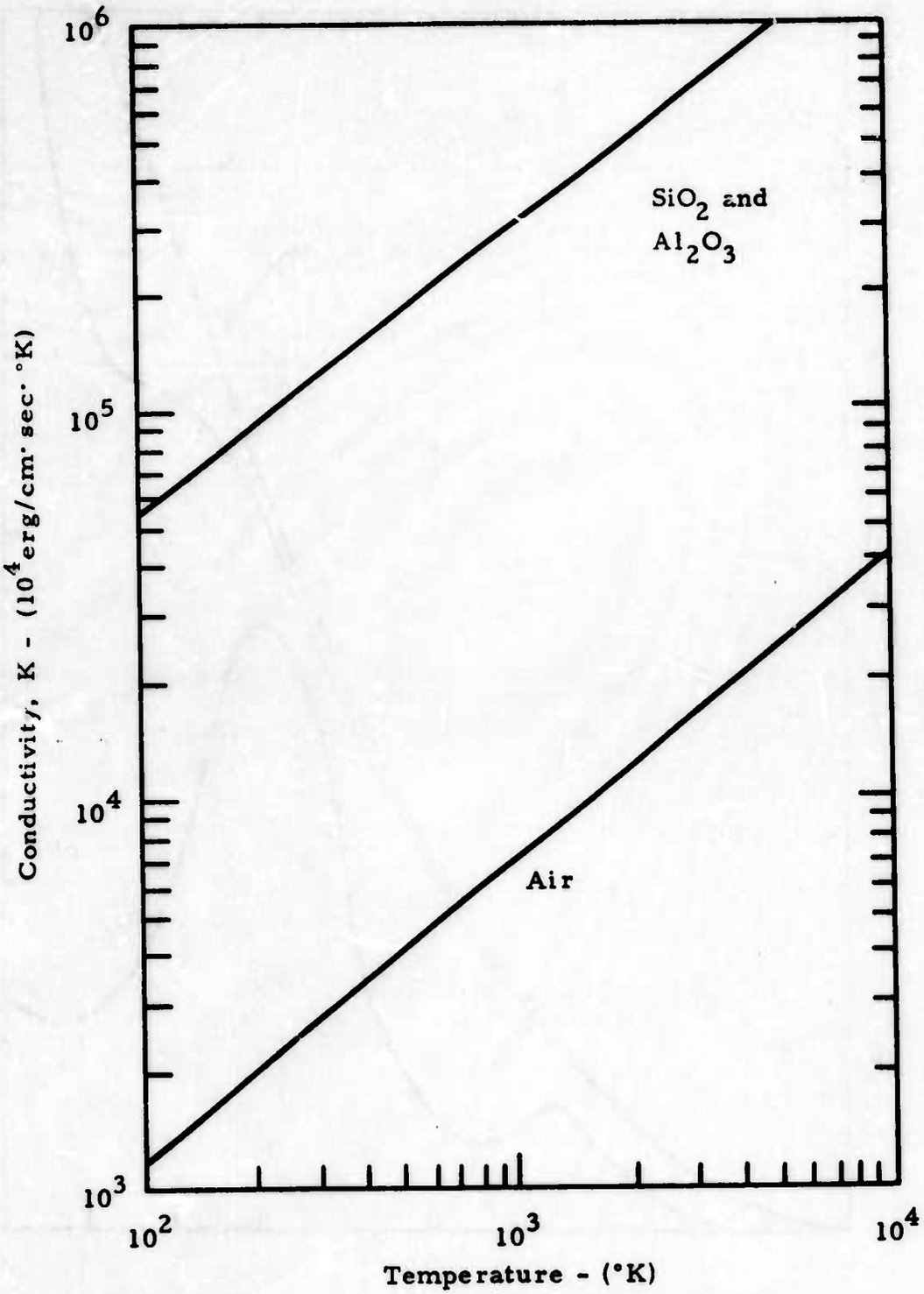


Figure 14. Thermal Conductivity of Various Material from Equation (1.0).

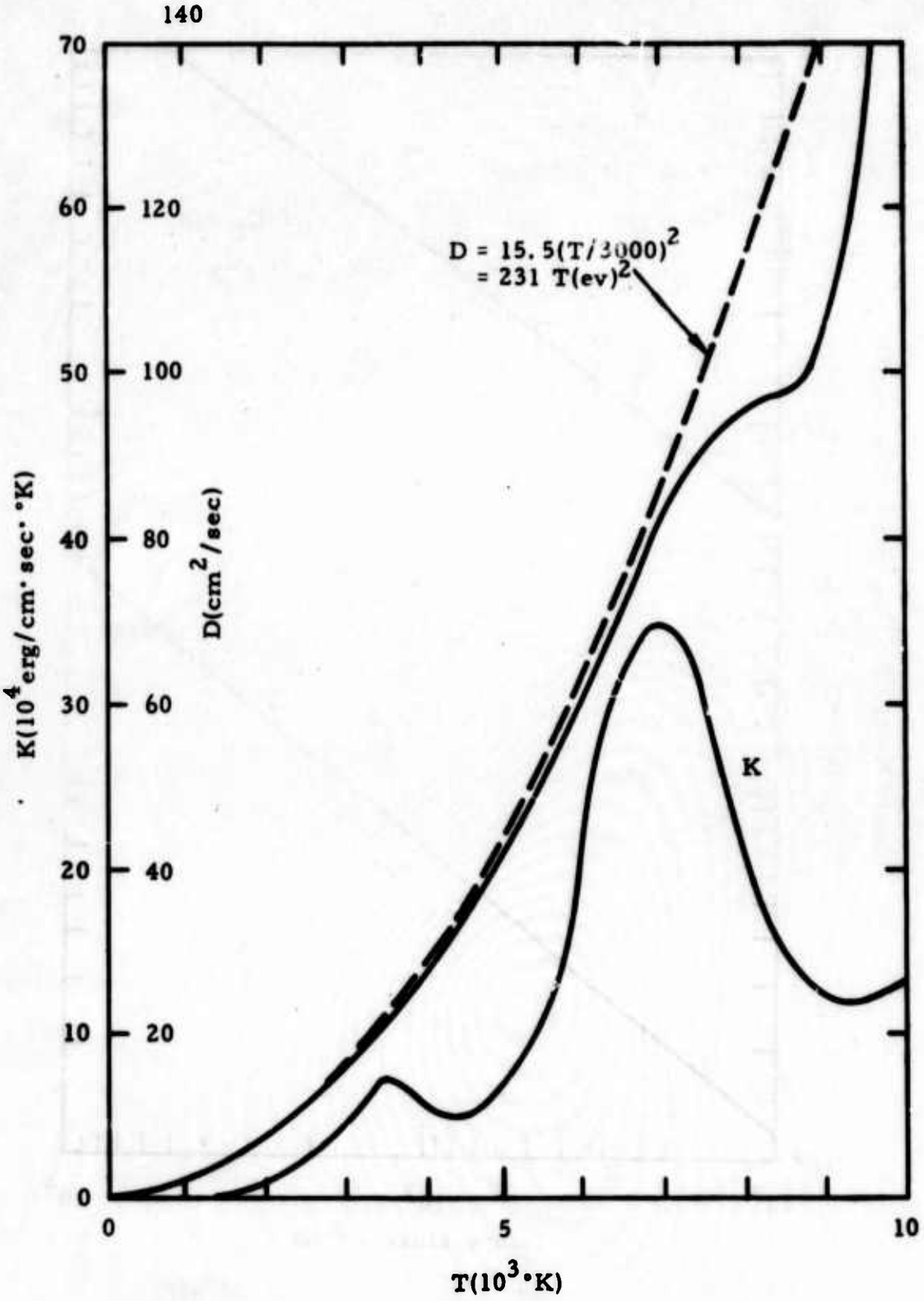


Figure 15. Air Conductivity, K, and Diffusivity, D, at One Atmosphere Pressure.

$$\chi_e = \frac{3}{2} N_e k D_e \tag{121}$$

where

$$D_e = \frac{1}{3} \bar{v} \lambda_e \tag{122}$$

is the electron diffusion coefficient, $\bar{v}$ is the average electron velocity and λ_e is the mean free path. In terms of the collision frequency, the temperature-dependent diffusion coefficient is given by

$$D_e = \frac{kT_e}{m\nu_c} \tag{123}$$

Ambipolar diffusion is discussed in Section III-2.

4. THERMIONIC EMISSION

The thermionic emission (Ref. 38) from a material at high temperatures can be evaluated by assuming that the material emitted outside of the surface is in equilibrium with the material inside. Using the Fermi distribution for the electrons,

$$f(\epsilon + \epsilon_K) = \frac{1}{1 + e^{(W+\epsilon_K)/kT}} \tag{124}$$

where ϵ is the energy required to remove an electron from some reference energy level to a state of rest outside the metal, ϵ_K is the kinetic energy of an electron outside of the metal, and W is the work function. Multiplying equation (124) by $N_e v$, assuming that $(W+\epsilon_K) >> kT$, and integrating over the density of states in phase space gives

$$J = \frac{m}{2\pi^2 h^3} (kT)^2 e^{-W/kT}$$

$$= 7.72 \times 10^{20} T^2 e^{-W/kT} \text{ cm}^{-2} - \text{sec}^{-1} \qquad (125)$$

This is just the well known Richardson equation for thermionic emission.

On the basis of the derivation given above, one expects that the flux of electrons for different materials would depend on the density of electrons in the conduction band. Thus, to some degree, the difference in the conductivity between materials should indicate the difference to be expected in the thermionic emission. The conductivity of most metals is about 10^7 mhos/meter while the conductivity for quartz is $\sim 10^{-17}$ mhos/meter. Thus, one would expect the thermionic emission for quartz to be down several orders of magnitude from that given by the Richardson equation.

However, experimental evidence (Ref. 39) indicates that there is a substantial flux of electrons from quartz and borosilicate targets. This flux has been interpreted as being due to heating of surface contaminants. This effect would show up in the Richardson equation as a change in the constant factor. Since it is not clear, without further study, what value the constant factor should have for SiO_2 and Al_2O_3, the value given in equation (125) has been used for the preliminary calculations given in this report. This provides an upper limit on the flux of electrons due to thermionic emission.

5. MACROMOLECULAR COMPLEXES

A further source of electrons may result from the interaction of a laser beam with naturally occuring molecular clusters. Such clusters generally have a substantially lower ionization potential. Of primary importance would be water clusters in the atmosphere. The evaluation of the density, size, and ionization potential of such clusters is a difficult problem for which there exists little information. The problem is neglected in the present report.

SECTION V

RESULTS

Calculations have been carried out to determine the effect of impurities on the breakdown of air by a CO_2 laser beam. The particular impurities considered were sapphire (Al_2O_3) and quartz (SiO_2), although the CELAB hydrodynamic code developed for this study is quite general and can be utilized for the study of other materials.

For numerical considerations in the present report, the only particle sizes considered were 12, 24, 48 and 96 μ, although the code is capable of handling particles of any size. The code divides the particles into a number of zones, each zone having a width of approximately 10 μ. In air, immediately outside the particle, the zone width is increased. The hydrodynamic time steps are of the order of 10^{-10} seconds. The temperatures, both ionic and electronic, are considered throughout the particle and into the air.

Figure 16 shows the calculated electron density in the first air zone as a function of the time for four different sizes of Al_2O_3 in the field of a 10.6 μ laser beam having an intensity of 3×10^8 watts/cm^2. The peak electron density is for the largest particle (96 μ), however, the difference in the electron density for the various sizes is not very significant. The electron density in the first zone reaches the steady state value of about 8×10^{16} electrons/cm^3. This represents the degree of ionization of about 0.3%.

A similar pair of curves for SiO_2 and sizes 24 and 48 μ is shown in figure 17. Again, the laser is 10.6 μ with an intensity of 3×10^8 watts/cm^2. A steady-state value of the electron density in the first zone is apparently obtained at about 1.5×10^{-7} seconds.

The curves for SiO_2, figure 17, show a pronounced maximum and minimum occurring at approximately 3.5 and 6×10^{-8} seconds. Again, the

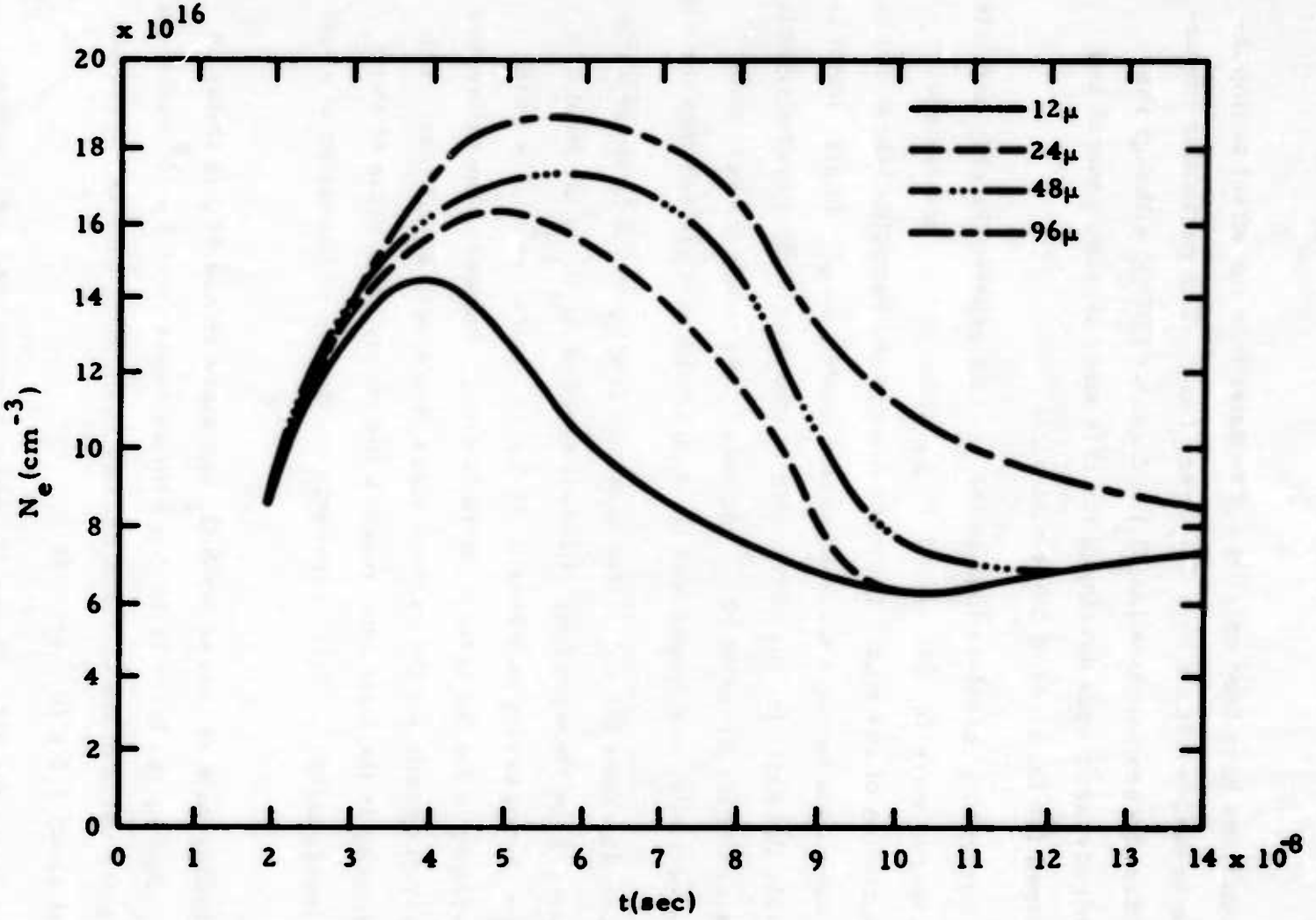


Figure 16. Electron Density vs Time for Different Size Al_2O_3 Particles in the Beam CO_2 Laser at an Intensity of 3×10^8 watts/cm^2.

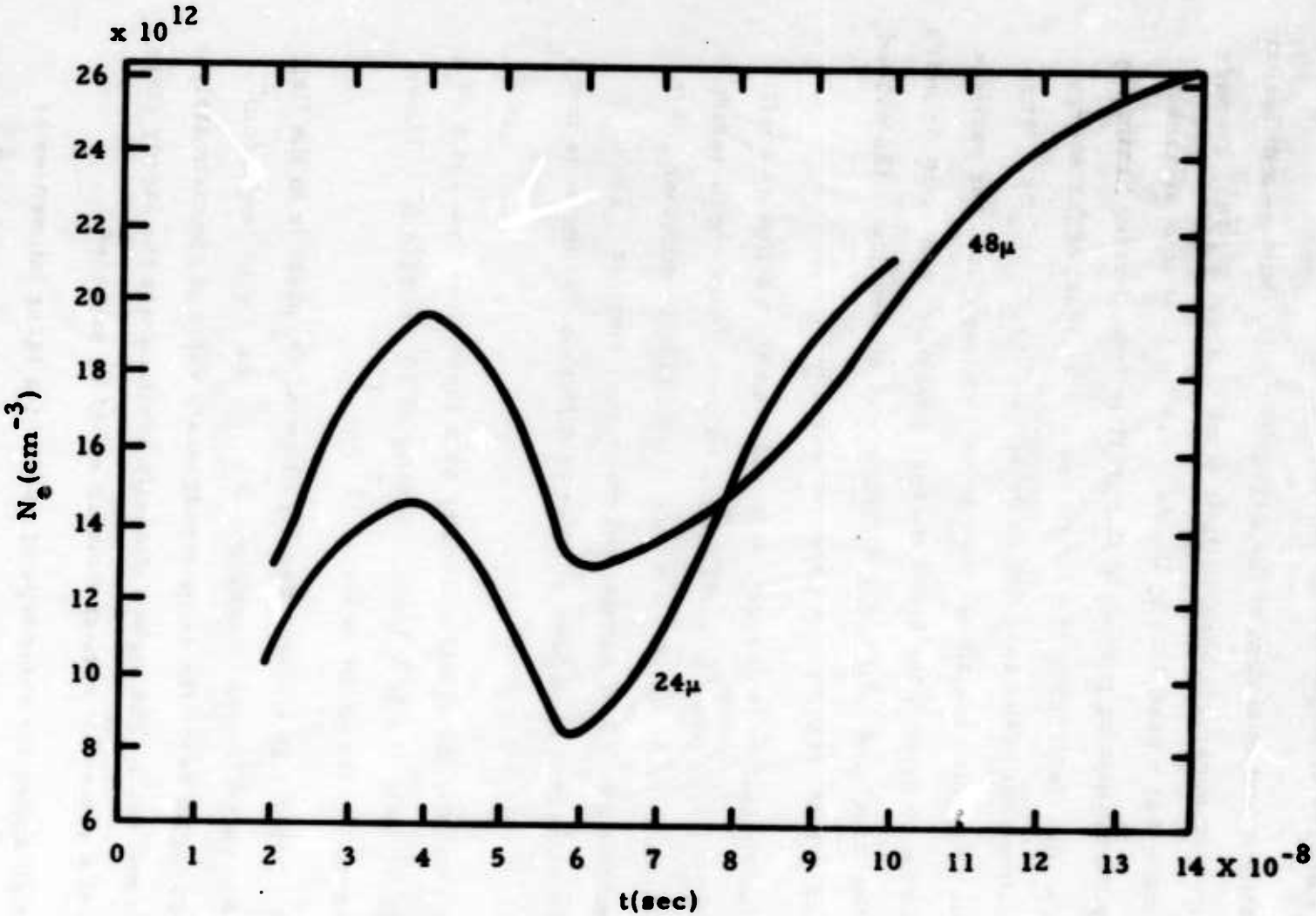


Figure 17. Electron Density vs Time for Different Size SiO_2 Particles in the Beam CO_2 Laser at an Intensity of 3×10^8 watts/cm^2.

difference in the electron density for the two sizes is not substantial. However, the difference in the peak electron density for SiO_2 as compared to Al_2O_3 is about 4 orders of magnitude.

The maximum and minimum of the curves for SiO_2 indicate a diffusion type of motion with an apparent characteristic time of about 3×10^{-8} seconds. The curves indicate that a peak density builds up in the first zone at some point, shielding the remaining portion of the particle from further heating by the laser beam as the electrons in the first zone absorb most of the energy. The electrons diffuse out of the zone with a relatively large velocity. When the electron loss from the zone approaches some limiting value, the particle again begins to absorb most of the laser energy, heating up and emitting more electrons into the first zone. Since the diffusion has slowed down, the second peak is more persistent, leading to a quasi-steady state.

If this interpretation is correct, it would indicate a diffusion coefficient of approximately $10^2 cm^2/seconds$. This value is more characteristic of the free electron diffusion coefficient than the ambipolar diffusion coefficient which would be about 2 to 3 orders of magnitude smaller. Thus, it appears that the electrons have been allowed to diffuse out of the zone much too rapidly.

Figure 18 shows the electron density as a function of time for a 24 μ and 48 μ quartz particle at a CO_2 laser intensity of 10^8 watts/cm^2. These curves have a growth rate on the order of 10^7 sec^{-1}.

Figure 19 shows an electron density of an Al_2O_3 particle in the field of a CO_2 laser at two different intensities, 3×10^7 and 1×10^7 watts/cm^2. Both curves appear to reach the same steady-state value of approximately 3×10^{17} electrons/cm^2. The higher intensity beam causes the steady state to be reached at a somewhat earlier time as would be expected.

Figure 20 shows the same type of curves for laser intensities of 3×10^8 and 1×10^8 watts/cm^2. The higher intensity again causes the peak

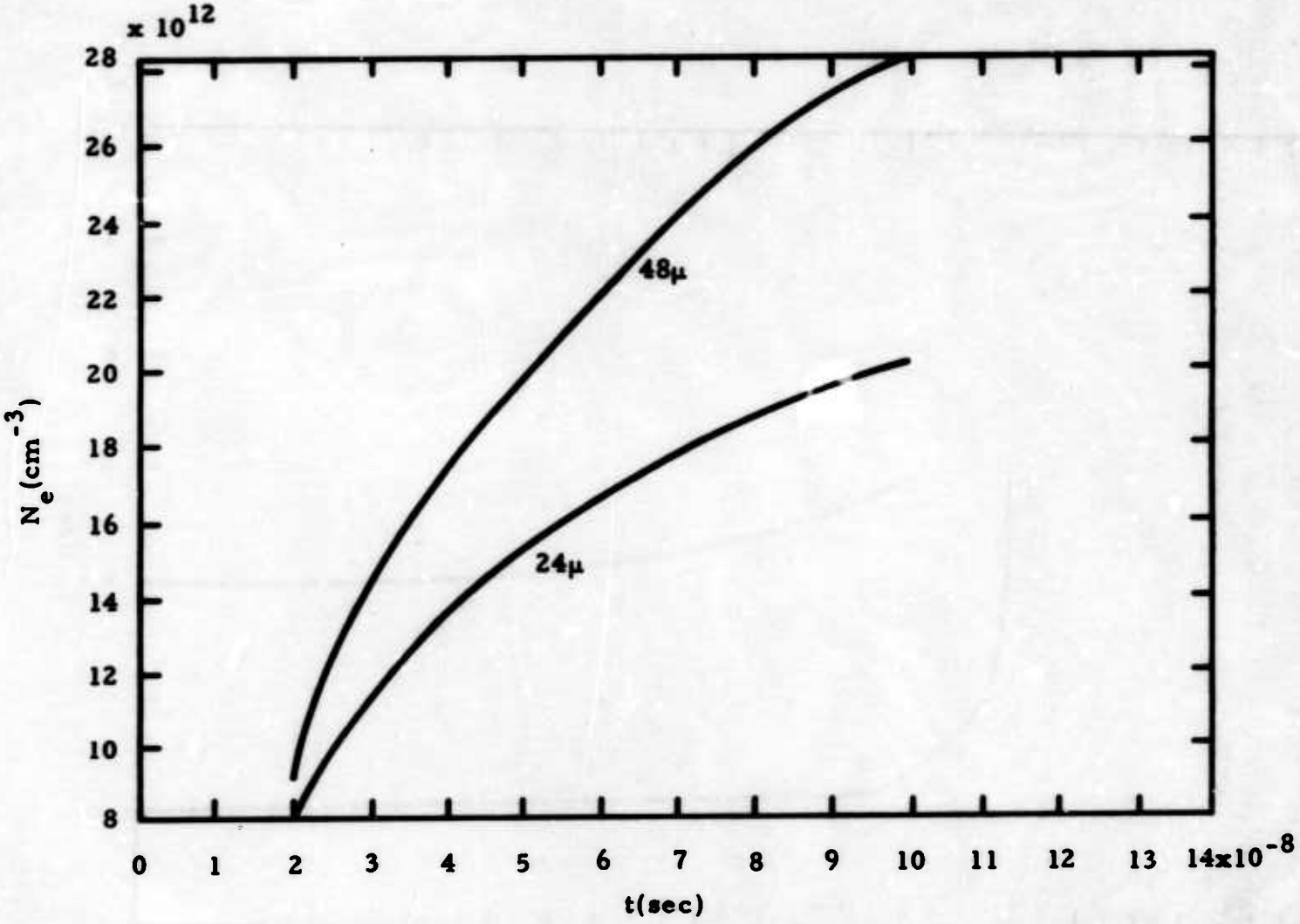


Figure 18. Electron Density vs Time for Different Size SiO_2 Particles in the Beam CO_2 Laser in an Intensity of 1×10^8 watts/cm^2.

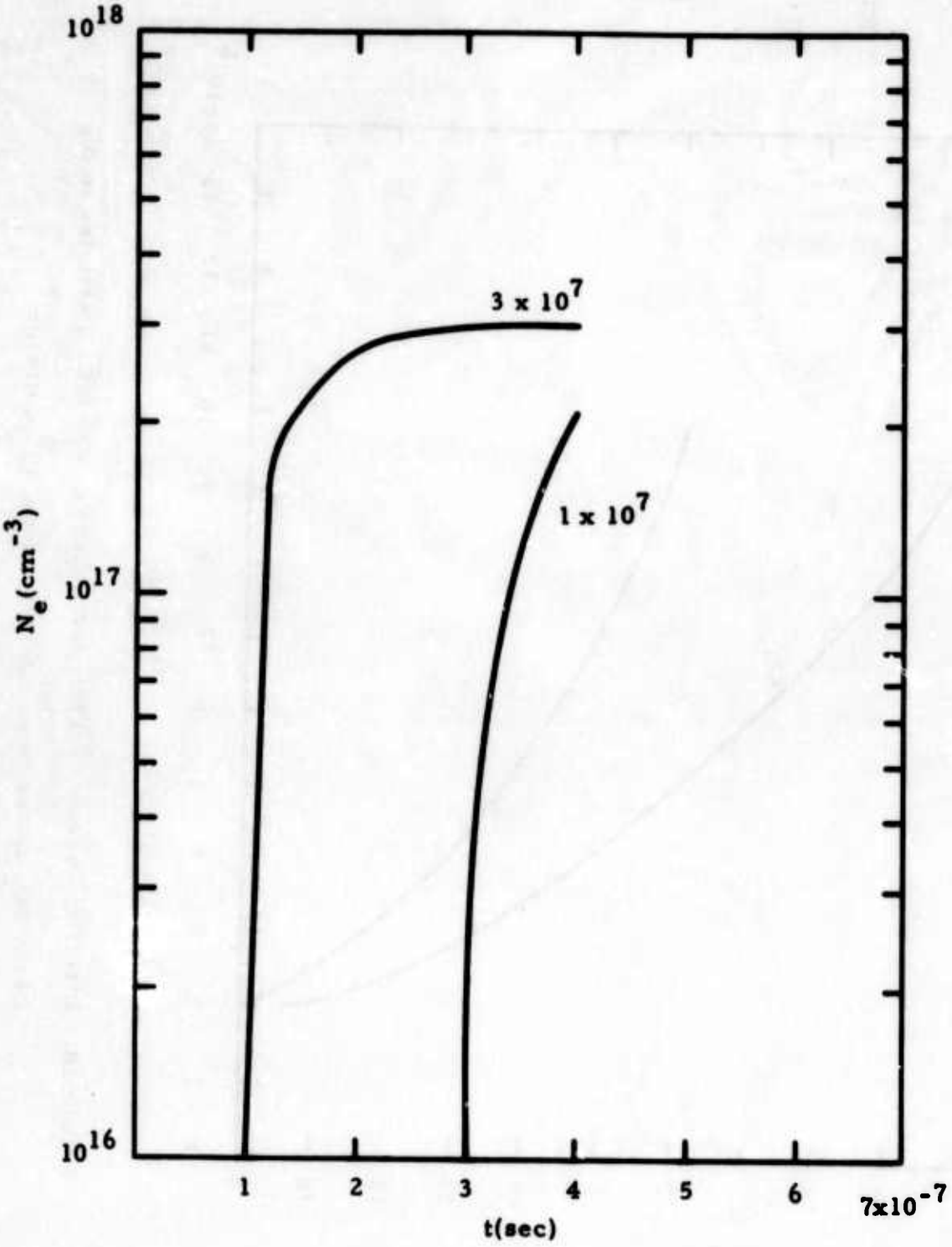


Figure 19. Electron Density vs Time for a 48μ Al_2O_3 Particle in the Beam of a CO_2 Laser at Two Different Intensities.

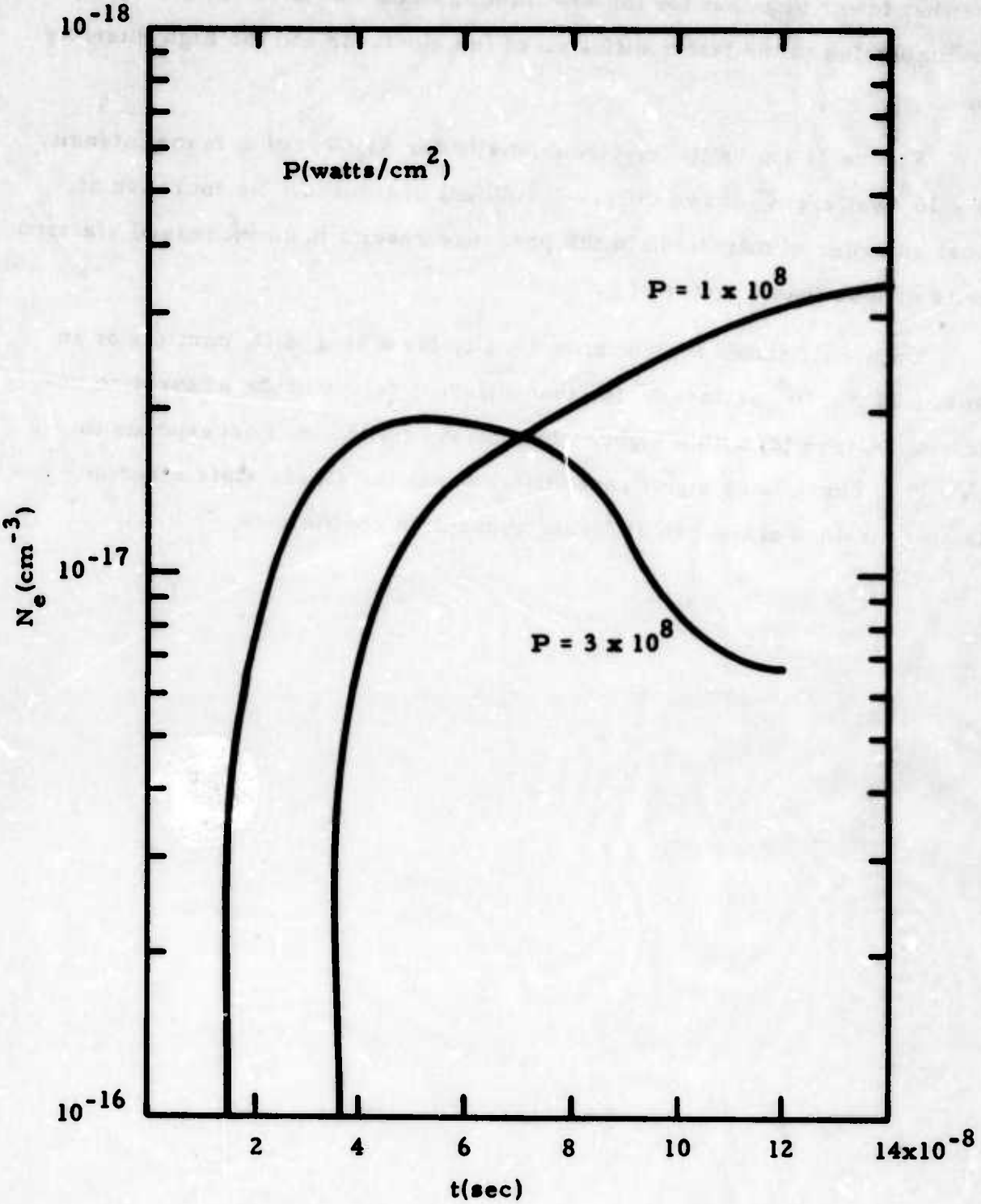


Figure 20. Electron Density vs Time for a 48μ Al_2O_3 Particle in the Beam of a CO_2 Laser at Two Different Intensities.

electron density to be attained at an earlier time, however, the density is somewhat lower than that for the low intensity beam at later times. This is presumably due to the faster diffusion of the electrons and the high intensity beam.

Figure 21 shows the electron density for Al_2O_3 with a laser intensity of 3×10^8 watts/cm^2 at two different ambient pressures. An increase of almost an order of magnitude in the pressure results in an increased electron density of less than a factor of 2.

Figure 22 shows the electron density for a 48 μ SiO_2 particle at an intensity of 3×10^8 watts/cm^2 for two different values of the absorption coefficient. Curve (2) with a higher absorption coefficient, corresponds to figure 17. There is no significant difference in the steady state electron densities attained at the two different absorption coefficients.

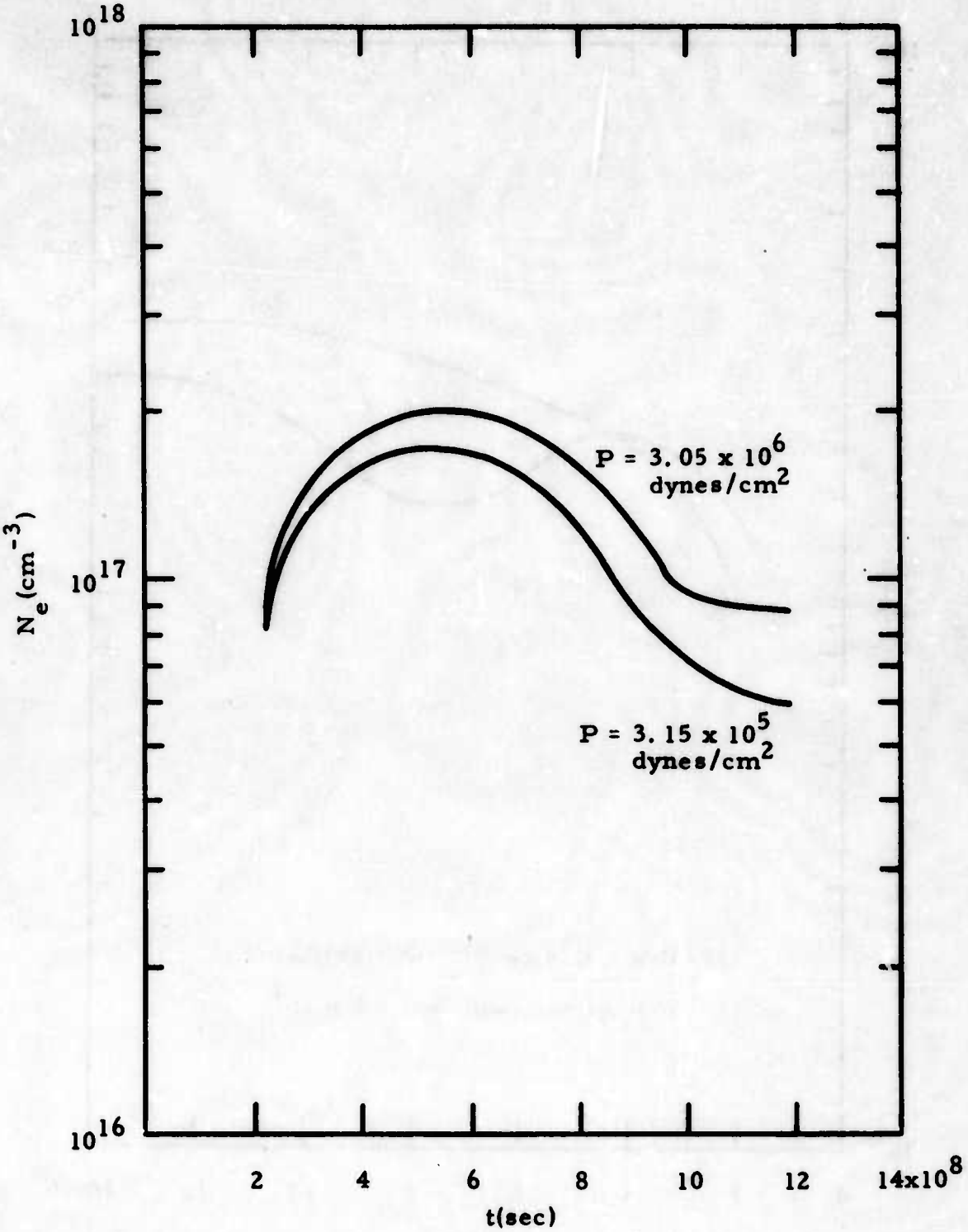


Figure 21. Electron Density vs Time for a 48μ Al_2O_3 Particle in the Field of a CO_2 Laser at an Intensity of 3 x 10^8 watts/cm^2 at Different Ambient Pressures.

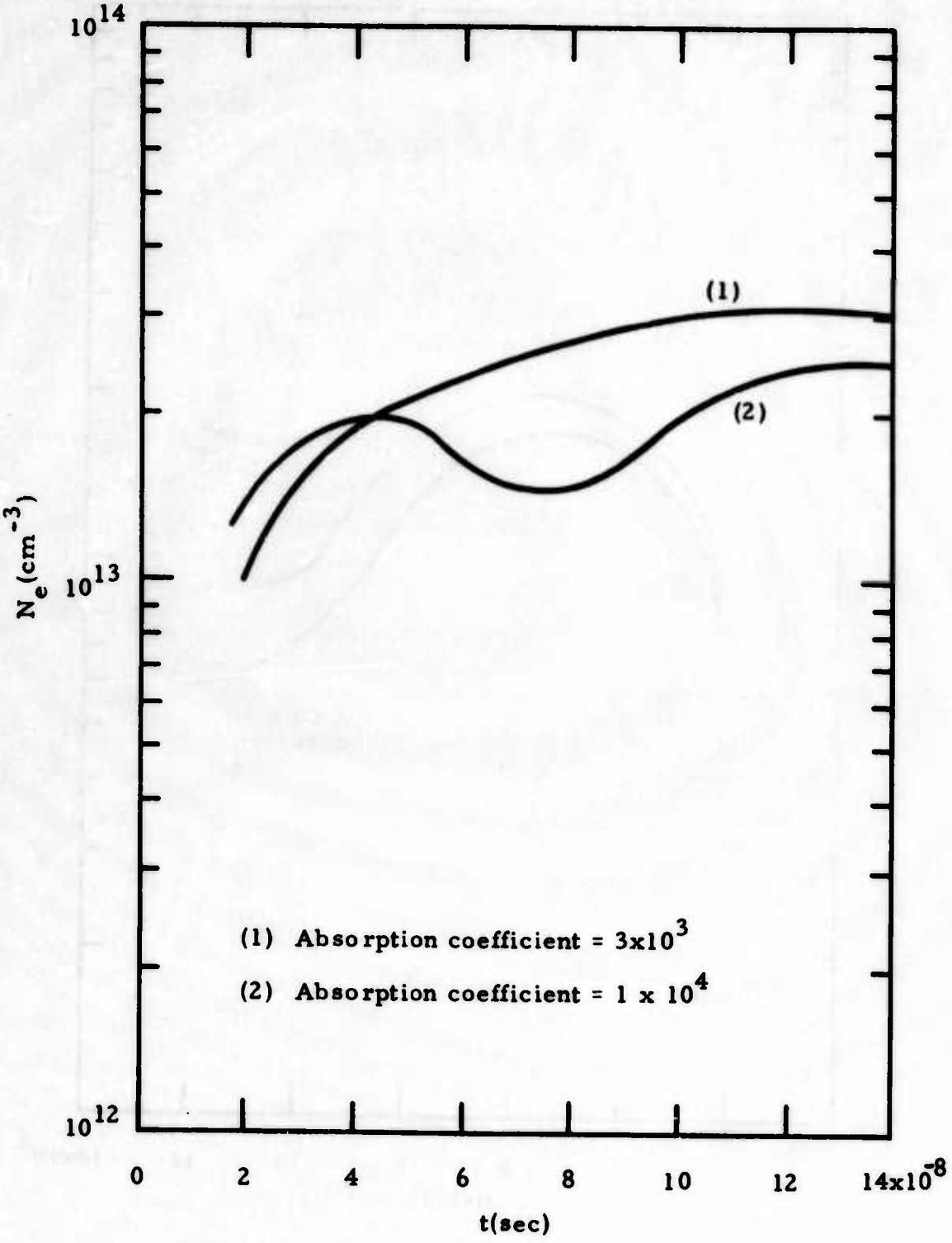


Figure 22. Electron Density vs Time for a 48μ SiO_2 Particle in the Field of a CO_2 Laser at an Intensity of 3×10^8 watts/cm^2 for Different Values of the SiO_2 Absorption Coefficient.

SECTION VI

CONCLUSIONS

It was shown in the theoretical analysis of the breakdown of air that the decrease in power required for breakdown is directly proportional to the increase in the total collision frequency. As the electron density is increased the total collision frequency is increased through electron-ion interaction. After the degree of ionization is increased beyond $\sim 10^{-3}$, the electron-ion interaction becomes dominant and the rate of energy absorption increases rapidly, leading to breakdown.

The results of the calculations performed for this report show that the atmospheric impurities are a source for a substantial density of free electrons as suggested in references 1 and 2. This is especially true for Al_2O_3.

The temperatures calculated for the electrons were relatively low, about 10^3°K, since the energy absorption was just compensated by the loss of electron energy to molecular vibration. Thus, the electron densities calculated were from the impurity particles and not from the atmospheric N_2 or O_2. Under these conditions one cannot draw any conclusions about the relative effectiveness of the impurity particles in breaking down the atmosphere.

The electron diffusion coefficient used in the calculations was too large, being more consistent with the value for the free electron diffusion than the ambipolar diffusion. This would allow the electron density to decrease much too rapidly leading to the relatively low temperatures mentioned previously. On the otherhand, the use of the Richardson equation to describe the thermionic emission for Al_2O_3 may have given too large an electron flux. These two possible overestimates tend to compensate each other; however, the large electron diffusion coefficient may lead to the more serious discrepancy. The electron density calculated for Al_2O_3 was

approaching values that could lead to substantial changes in the total collision frequency, thus, this area requires more study.

It was found during the course of this investigation that information on the material properties of atmospheric impurities was very difficult to find. Further studies of air breakdown should lead to a great deal of useful information on experimental measurements that are needed. An example of this was indicated by the lack of availability of information on the type of molecules formed by Al_2O_3 in the vapor phase.

The CELAB hydrodynamic code developed for this contract should be very useful for future investigations of the effectiveness of other impurities as a source of free electrons. It was originally intended that the present investigation would include calculations of the electron density resulting from clusters on macromolecules such as water vapor clusters. It is expected that the lower ionization potentials of such clusters might be a major source of electrons. Time, however, did not permit such studies at the present time. It is suggested that this potential source of electrons be investigated in the future along with other impurities existing in the atmosphere.

A second approach to the calculation of atmospheric breakdown was started but remained unfinished as the present investigation was terminated. In this approach a Monte Carlo calculation was used to follow the time dependent development of the electron distribution function including the effects of ionization. While the development of the code was not finished, enough progress had been made to justify its inclusion as an appendix in this report so that anyone wishing to follow this course of investigation could utilize the results obtained thus far.

APPENDIX I

DESCRIPTION OF CELAB CODE

This appendix gives a description of the difference methods used in the code CELAB. CELAB is a one-dimensional Lagrangian code which allows plane, cylindrical or spherical geometry through an input parameter $\alpha = 1, 2, 3$. We begin with a description of the difference equation approximation to the hydrodynamic equations. An implicit scheme is used to solve the energy equation including the thermal conductivity term. The method for control of the fundamental time step is given and the energy deposition is discussed.

1. DIFFERENCING FOR CELAB

The differential equations to be solved are:

Conservation of Momentum:

$$\frac{\partial U}{\partial t} = - R^{\alpha - 1} \frac{\partial P + Q}{\partial M} \tag{126}$$

Definition of Velocity:

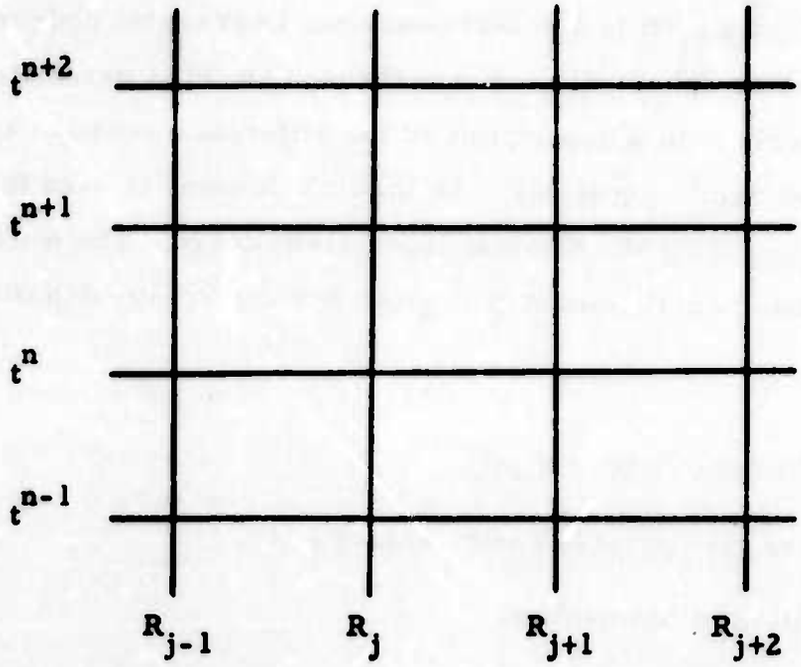
$$\frac{\partial R}{\partial t} = U \tag{127}$$

Conservation of Mass:

$$V = \frac{1}{D} = \frac{1}{\alpha} \frac{\partial R^{\alpha}}{\partial M} \tag{128}$$

Conservation of Energy:

$$\frac{\partial E}{\partial t} = - P \frac{\partial V}{\partial t} + \varphi \overline{K} + \frac{\partial}{\partial M}\left[\chi R^{\alpha-1} \frac{\partial T}{\partial R}\right] \qquad (129)$$



α = dimension of space

R = coordinate

U = velocity

M = mass

V = specific volume

D = density

E = specific energy

P = pressure

φ = laser flux

$\overline{K}$ = absorption coefficient

χ = thermal conductivity

T = temperature

Equation (126) can be differenced directly to give

$$U_j^{n+\frac{1}{2}} = U_j^{n-\frac{1}{2}} - (R_j^n)^{\alpha-1} \frac{P_{j+\frac{1}{2}}^n - P_{j-\frac{1}{2}}^n}{M_{j+\frac{1}{2}} + M_{j-\frac{1}{2}}} \cdot 2\Delta t^n \tag{130}$$

where

$$\Delta t^n = \tfrac{1}{2}(t^{n+1} - t^{n-1}) \tag{131}$$

The second equation gives

$$R_j^{n+1} = R_j^n + U_j^{n+\frac{1}{2}} \Delta t^{n+\frac{1}{2}} \tag{132}$$

with

$$\Delta t^{n+\frac{1}{2}} = t^{n+1} - t^n \tag{133}$$

The conservation of mass becomes

$$V_{j-\frac{1}{2}}^{n+1} = \frac{1}{\alpha} \frac{(R_j^{n+1})^{\alpha} - (R_{j-1}^{n+1})^{\alpha}}{M_{j-\frac{1}{2}}} \tag{134}$$

or

$$D_{j-\frac{1}{2}}^{n+1} = \frac{\alpha M_{j-\frac{1}{2}}}{(R_j^{n+1})^{\alpha} - (R_{j-1}^{n+1})^{\alpha}} \tag{135}$$

For the energy equation, we use an implicit scheme to solve for the new energy and pressure, including the thermal conductivity terms. The left side of the energy equation is differenced directly

$$\left(\frac{\partial E}{\partial t}\right)^{n+\frac{1}{2}}_{j-\frac{1}{2}} = \frac{W^{n+\frac{1}{2}}_{j-\frac{1}{2}}}{\Delta t^{n+\frac{1}{2}}} \tag{136}$$

where

$$W^{n+\frac{1}{2}}_{j-\frac{1}{2}} = E^{n+1}_{j-\frac{1}{2}} - E^{n}_{j-\frac{1}{2}} \tag{137}$$

To compute the $P \cdot dV$ work implicitly, we assume that the pressure is a linear function of the internal energy

$$P = P_H + \Gamma D E \tag{138}$$

where Γ is the Grüneisen coefficient and P_H is the Hugoniot pressure which is a function of density only. For an ideal gas, $P_H = 0$ and $\Gamma = \gamma - 1$.

Including the artificial viscosity, the $P \cdot dV$ term gives

$$(P \cdot dV)^{n+\frac{1}{2}}_{j-\frac{1}{2}} = (\Gamma D^{n+1}_{j-\frac{1}{2}} E^{n+1}_{j-\frac{1}{2}} + P^{n+1}_{H_{j-\frac{1}{2}}} + P^{n}_{j-\frac{1}{2}} + 2Q^{n+\frac{1}{2}}_{j-\frac{1}{2}}) \cdot \frac{\Delta V^{n+\frac{1}{2}}_{j-\frac{1}{2}}}{2} \tag{139}$$

where

$$\Delta V^{n+\frac{1}{2}}_{j-\frac{1}{2}} = V^{n+1}_{j-\frac{1}{2}} - V^{n}_{j-\frac{1}{2}} \tag{140}$$

To avoid numerical cancellation in computing this difference, we use the relation

$$\Delta V^{n+\frac{1}{2}}_{j-\frac{1}{2}} = \left(\frac{\partial V}{\partial t}\right)^{n+\frac{1}{2}}_{j-\frac{1}{2}} \cdot \Delta t^{n+\frac{1}{2}} \tag{141}$$

and the conservation of mass to write

$$\Delta V_{j-\frac{1}{2}}^{n+\frac{1}{2}} = \frac{(R_j^{n+\frac{1}{2}})^{\alpha-1} U_j^{n+\frac{1}{2}} - (R_{j-1}^{n+\frac{1}{2}})^{\alpha-1} U_{j-1}^{n+\frac{1}{2}}}{M_{j-\frac{1}{2}}} \cdot \Delta t^{n+\frac{1}{2}} \quad (142)$$

For the laser deposition we write,

$$(\varphi\overline{K})_{j-\frac{1}{2}}^{n+\frac{1}{2}} = F_{j-\frac{1}{2}}^{n+\frac{1}{2}} \quad (143)$$

where $F_{j-\frac{1}{2}}$ will be computed using Bier's law and an appropriate absorption coefficient.

For the thermal conductivity term of the energy equation, we assume a power law for the temperature dependence of the conductivity,

$$\chi(T) = \chi_0 T^{\beta} \quad (144)$$

where χ_0 and β depend on material and may be slowly varying functions of temperature and density. We further assume that the relation between temperature and internal energy is linear

$$E - E_0 = C(T - T_0) \quad (145)$$

where C is a specific heat and E_0 is the internal energy at a reference temperature T_0. Let

$$E_1 = E_0 - CT_0$$

so that

$$T = \frac{1}{C}(E - E_1) \quad (146)$$

The conductivity term

$$\dot{E}_c = \frac{\partial}{\partial M}\left[\chi(T) R^{\alpha-1} \frac{\partial T}{\partial R}\right] \tag{147}$$

can be differenced implicitly,

$$(\dot{E}_c)^{n+\frac{1}{2}}_{j-\frac{1}{2}} = \frac{1}{M_{j-\frac{1}{2}}} \left\{(R_j^{n+\frac{1}{2}})^{\alpha-1}\left[\chi \frac{\partial T}{\partial R}\right]^{n+\frac{1}{2}}_j - (R^{n+\frac{1}{2}}_{j-1})^{\alpha-1}\left[\chi \frac{\partial T}{\partial R}\right]^{n+\frac{1}{2}}_{j-1}\right\} \tag{148}$$

The direct method of determining the energy flow at the zone boundary

$$\left[\chi(T) \frac{\partial T}{\partial R}\right]^{n+\frac{1}{2}}_j$$

is to use the conservation of energy

$$\left[\chi(T) \frac{\partial T}{\partial R}\right]_{j-} = \left[\chi(T) \frac{\partial T}{\partial R}\right]_{j+} \tag{149}$$

and the approximation

$$\chi_{j\pm} = \chi_{j\pm\frac{1}{2}}$$

to solve for the temperature at the zone boundary,

$$T_j^{n+\frac{1}{2}} = \frac{X^{n+\frac{1}{2}}_{j-\frac{1}{2}} T^{n+\frac{1}{2}}_{j-\frac{1}{2}} + X^{n+\frac{1}{2}}_{j+\frac{1}{2}} T^{n+\frac{1}{2}}_{j+\frac{1}{2}}}{X^{n+\frac{1}{2}}_{j-\frac{1}{2}} + X^{n+\frac{1}{2}}_{j+\frac{1}{2}}} \tag{150}$$

where

$$X^{n+\frac{1}{2}}_{j-\frac{1}{2}} = \frac{\chi^{n+\frac{1}{2}}_{j-\frac{1}{2}}}{R_j^{n+\frac{1}{2}} - R^{n+\frac{1}{2}}_{j-1}} \tag{151}$$

This temperature can then be used to compute the energy flow at the zone boundary.

$$\left[\chi \frac{\partial T}{\partial R}\right]_j^{n+\frac{1}{2}} = \frac{2X_{j-\frac{1}{2}}^{n+\frac{1}{2}} X_{j+\frac{1}{2}}^{n+\frac{1}{2}} (T_{j+\frac{1}{2}}^{n+\frac{1}{2}} - T_{j-\frac{1}{2}}^{n+\frac{1}{2}})}{X_{j-\frac{1}{2}}^{n+\frac{1}{2}} + X_{j+\frac{1}{2}}^{n+\frac{1}{2}}}$$

Using equations (144) and (146), the implicit dependence of $T_{j-\frac{1}{2}}^{n+\frac{1}{2}}$ and $\chi_{j-\frac{1}{2}}^{n+\frac{1}{2}}$ on the energy differences can be determined

$$T_{j-\frac{1}{2}}^{n+\frac{1}{2}} = T_{j-\frac{1}{2}}^{n} + \frac{1}{2C_{j-\frac{1}{2}}} W_{j-\frac{1}{2}}^{n+\frac{1}{2}} \tag{151}$$

$$\chi_{j-\frac{1}{2}}^{n+\frac{1}{2}} = \chi_{j-\frac{1}{2}}^{n} \left\{1 + \frac{\beta_{j-\frac{1}{2}}}{2C_{j-\frac{1}{2}} T_{j-\frac{1}{2}}^{n}} W_{j-\frac{1}{2}}^{n+\frac{1}{2}}\right\} \tag{152}$$

Then to first order in the energy differences, the energy flow at the zone boundary is

$$\left[\chi \frac{\partial T}{\partial R}\right]_j^{n+\frac{1}{2}} = \frac{2(T_{j+\frac{1}{2}}^{n} - T_{j-\frac{1}{2}}^{n}) + W_{j+\frac{1}{2}}^{n+\frac{1}{2}}/C_{j+\frac{1}{2}} - W_{j-\frac{1}{2}}^{n+\frac{1}{2}}/C_{j-\frac{1}{2}}}{Y_{j-\frac{1}{2}}^{n}\left[1 - \beta_{j-\frac{1}{2}} W_{j-\frac{1}{2}}^{n+\frac{1}{2}}/2C_{j-\frac{1}{2}} T_{j-\frac{1}{2}}^{n}\right] + Y_{j+\frac{1}{2}}^{n}\left[1 - \beta_{j+\frac{1}{2}} W_{j+\frac{1}{2}}^{n+\frac{1}{2}}/2C_{j+\frac{1}{2}} T_{j+\frac{1}{2}}^{n}\right]} \tag{153}$$

where

$$Y_{j-\frac{1}{2}}^{n} = \frac{R_j^{n+\frac{1}{2}} - R_{j-1}^{n+\frac{1}{2}}}{\chi_{j-\frac{1}{2}}^{n}} \tag{154}$$

Expanding to first order in the energy differences and collecting terms we obtain

$$\left[\chi \frac{\partial T}{\partial R}\right]_j^{n+\frac{1}{2}} = \frac{1}{Y_{j-\frac{1}{2}}^n + Y_{j+\frac{1}{2}}^n} \left\{ 2\left(T_{j+\frac{1}{2}}^n - T_{j-\frac{1}{2}}^n\right) \right.$$

$$+ \left[1. + \frac{T_{j+\frac{1}{2}}^n - T_{j-\frac{1}{2}}^n}{Y_{j-\frac{1}{2}}^n + Y_{j+\frac{1}{2}}^n} \cdot \frac{\beta_{j+\frac{1}{2}} Y_{j+\frac{1}{2}}^n}{T_{j+\frac{1}{2}}^n} \right] \frac{W_{j+\frac{1}{2}}^{n+\frac{1}{2}}}{C_{j+\frac{1}{2}}}$$

$$\left. - \left[1 - \frac{T_{j+\frac{1}{2}}^n - T_{j-\frac{1}{2}}^n}{Y_{j-\frac{1}{2}}^n + Y_{j+\frac{1}{2}}^n} \cdot \frac{\beta_{j-\frac{1}{2}} Y_{j-\frac{1}{2}}^n}{T_{j-\frac{1}{2}}^n} \right] \frac{W_{j-\frac{1}{2}}^{n+\frac{1}{2}}}{C_{j-\frac{1}{2}}} \right\} \qquad (155)$$

Combining this result with equation(148) and using the notation

$$Z_j^n = \frac{(R_j^{n+\frac{1}{2}})^{\alpha-1}}{Y_{j-\frac{1}{2}}^n + Y_{j+\frac{1}{2}}^n} \qquad (156)$$

gives an expression for the heat conductivity of the form

$$(\dot{E}_c)_{j-\frac{1}{2}}^{n+\frac{1}{2}} = \frac{1}{M_{j-\frac{1}{2}}} \left\{ 2Z_j^n (T_{j+\frac{1}{2}}^n - T_{j-\frac{1}{2}}^n) - 2Z_{j-1}^n (T_{j-\frac{1}{2}}^n - T_{j-\frac{3}{2}}^n) \right.$$

$$+ \left(1 - \frac{T_{j-\frac{1}{2}}^n - T_{j-\frac{3}{2}}^n}{Y_{j-\frac{3}{2}}^n + Y_{j-\frac{1}{2}}^n} \cdot \frac{\beta_{j-\frac{3}{2}} Y_{j-\frac{3}{2}}^n}{T_{j-\frac{3}{2}}^n} \right) \cdot Z_{j-1}^n \cdot \frac{W_{j-\frac{3}{2}}^{n+\frac{1}{2}}}{C_{j-\frac{3}{2}}}$$

$$- \left[\left(1 - \frac{T_{j+\frac{1}{2}}^n - T_{j-\frac{1}{2}}^n}{Y_{j-\frac{1}{2}}^n + Y_{j+\frac{1}{2}}^n} \cdot \frac{\beta_{j-\frac{1}{2}} Y_{j-\frac{1}{2}}^n}{T_{j-\frac{3}{2}}^n} \right) Z_j^n \right.$$

$$+ \left(1 + \frac{T_{j-\frac{1}{2}}^n - T_{j-\frac{3}{2}}^n}{Y_{j-\frac{3}{2}}^n + Y_{j-\frac{1}{2}}^n} \cdot \frac{\beta_{j-\frac{1}{2}} Y_{j-\frac{1}{2}}^n}{T_{j-\frac{1}{2}}^n} \right) Z_{j-1}^n \left. \right] \frac{W_{j-\frac{1}{2}}^{n+\frac{1}{2}}}{C_{j-\frac{1}{2}}}$$

$$\left. + \left(1 + \frac{T_{j+\frac{1}{2}}^n - T_{j-\frac{1}{2}}^n}{Y_{j-\frac{1}{2}}^n + Y_{j+\frac{1}{2}}^n} \cdot \frac{\beta_{j+\frac{1}{2}} Y_{j+\frac{1}{2}}^n}{T_{j+\frac{1}{2}}^n} \right) Z_j^n \cdot \frac{W_{j+\frac{1}{2}}^{n+\frac{1}{2}}}{C_{j+\frac{1}{2}}} \right\} \qquad (157)$$

Combining equations(129), (136), (139), (143) and (157) yields an expression for the energy differences which can be written in the form

$$-A_j W^{n+\frac{1}{2}}_{j-\frac{3}{2}} + B_j W^{n+\frac{1}{2}}_{j-\frac{1}{2}} - C_j W^{n+\frac{1}{2}}_{j+\frac{1}{2}} = D_j \qquad (158)$$

where

$$A_j = \left(1 - \frac{T^n_{j-\frac{1}{2}} - T^n_{j-\frac{3}{2}}}{Y^n_{j-\frac{3}{2}} + Y^n_{j-\frac{1}{2}}} \cdot \frac{\beta_{j-\frac{3}{2}} Y^n_{j-\frac{3}{2}}}{T^n_{j-\frac{3}{2}}}\right) Z^n_{j-1} \cdot \frac{\Delta t^{n+\frac{1}{2}}}{M_{j-\frac{1}{2}} C_{j-\frac{3}{2}}} \qquad (159)$$

$$B_j = 1 + \tfrac{1}{2}\Gamma D^{n+1}_{j-\frac{1}{2}} \cdot \Delta V^{n+\frac{1}{2}}_{j-\frac{1}{2}} + \left[\left(1 - \frac{T^n_{j+\frac{1}{2}} - T^n_{j-\frac{1}{2}}}{Y^n_{j-\frac{1}{2}} - Y^n_{j+\frac{1}{2}}} \cdot \frac{\beta_{j-\frac{1}{2}} Y^n_{j-\frac{1}{2}}}{T^n_{j-\frac{1}{2}}}\right) \cdot Z^n_j + \left(1 + \frac{T^n_{j-\frac{1}{2}} - T^n_{j-\frac{3}{2}}}{Y^n_{j-\frac{3}{2}} - Y^n_{j-\frac{1}{2}}} \cdot \frac{\beta_{j-\frac{1}{2}} Y^n_{j-\frac{1}{2}}}{T^n_{j-\frac{1}{2}}}\right) Z^n_{j-1}\right] \cdot \frac{\Delta t^{n+\frac{1}{2}}}{M_{j-\frac{1}{2}} C_{j-\frac{1}{2}}} \qquad (160)$$

$$C_j = \left(1 + \frac{T^n_{j+\frac{1}{2}} - T^n_{j-\frac{1}{2}}}{Y^n_{j-\frac{1}{2}} + Y^n_{j+\frac{1}{2}}} \cdot \frac{\beta_{j+\frac{1}{2}} Y^n_{j+\frac{1}{2}}}{T^n_{j+\frac{1}{2}}}\right) Z^n_j \frac{\Delta t^{n+\frac{1}{2}}}{M_{j-\frac{1}{2}} C_{j+\frac{1}{2}}} \qquad (161)$$

$$D_j = F^{n+\frac{1}{2}}_{j-\frac{1}{2}} \Delta t^{n+\frac{1}{2}} - \left(\Gamma D^{n+1}_{j-\frac{1}{2}} E^n_{j-\frac{1}{2}} + P^{n+1}_{H_{j-\frac{1}{2}}} + P^n_{j-\frac{1}{2}} + 2Q^{n+\frac{1}{2}}_{j-\frac{1}{2}}\right) \frac{\Delta V^{n+\frac{1}{2}}_{j-\frac{1}{2}}}{2}$$

$$+ \left[Z^n_j (T^n_{j+\frac{1}{2}} - T^n_{j-\frac{1}{2}}) - Z^n_{j-1} (T^n_{j-\frac{1}{2}} - T^n_{j-\frac{3}{2}})\right] \cdot \frac{2\Delta t^{n+\frac{1}{2}}}{M_{j-\frac{1}{2}}} \qquad (162)$$

When the boundary condition at the origin

$$W^{n+\frac{1}{2}}_{-\frac{1}{2}} = 0 \qquad (163)$$

is used in equation (158) for j = 1, a linear relation between $W_{\frac{1}{2}}^{n+\frac{1}{2}}$ and $W_{\frac{3}{2}}^{n+\frac{1}{2}}$ is obtained. This linear relation can then be used in equation (158) for j = 2 to obtain a linear relation between $W_{\frac{3}{2}}^{n+\frac{1}{2}}$ and $W_{5/2}^{n+\frac{1}{2}}$. This process can be repeated throughout the mesh. The general form of the linear relation can be decuded in the following manner. Let

$$W_{j-\frac{1}{2}}^{n+\frac{1}{2}} = G_{j-\frac{1}{2}} W_{j+\frac{1}{2}}^{n+\frac{1}{2}} + H_{j-\frac{1}{2}} \tag{164}$$

Substituting this expression into equation (156) and collecting terms gives

$$[-A_j G_{j-\frac{3}{2}} G_{j-\frac{1}{2}} + B_j G_{j-\frac{1}{2}} - C_j] W_{j+\frac{1}{2}}^{n+\frac{1}{2}}$$

$$+ [-A_j G_{j-\frac{3}{2}} H_{j-\frac{1}{2}} - A_j H_{j-\frac{3}{2}} + B_j H_{j-\frac{1}{2}} - D_j] = 0$$

These relations are the desired equations when the equality is an identity, i.e., when

$$G_{j-\frac{1}{2}} = \frac{C_j}{B_j - A_j G_{j-\frac{3}{2}}} \tag{165}$$

$$H_{j-\frac{1}{2}} = \frac{D_j + A_j H_{j-\frac{3}{2}}}{B_j - A_j G_{j-\frac{3}{2}}} \tag{166}$$

These recursion relations along with the boundary condition at the origin,

$$G_{-\frac{1}{2}} = H_{-\frac{1}{2}} = 0 \tag{167}$$

can be used to determine the Gs and Hs by an outward sweep through the mesh. Then using the boundary condition at the outer boundary and equation (164), the energy differences $W_{j-\frac{1}{2}}^{n+\frac{1}{2}}$ can be determined by an inward sweep through the mesh. For the outer boundary condition, we take

$$W_{j+\frac{1}{2}}^{n+\frac{1}{2}} = 0 \qquad (168)$$

which corresponds to no conductivity across the outer boundary.

2. TIME STEP CONTROL

For stability of the hydrodynamic difference equations, we use an artificial viscosity similar to that discussed in Richtmeyer and Morton, and including a linear term in the velocity. If the material is in the solid state we use

$$Q = -D\,[C_0^2\,|\delta U| + C_1\,C_s]\cdot \delta U \qquad (169)$$

where $\delta U = \frac{\partial U}{\partial R}\delta R$, C_s is the sound velocity

$$C_s^2 = \left(\frac{\partial P}{\partial D}\right)_s \qquad (170)$$

and C_0 and C_1 are constants with typical values

$$C_0 = 2.0 \qquad C_1 = 0.25$$

The artificial viscosity in the form of equation(169) can exert a tension in a zone undergoing expansion, $\delta U > 0$. In the gas state a tensile artificial viscosity is not allowed and the modified form is used

$$Q = \begin{cases} -D\,[C_0^2\,|\delta U| + C_1\,C_s]\cdot \delta U & \delta U < 0 \\ 0 & \delta U > 0 \end{cases} \qquad (171)$$

Corresponding to the linear artificial viscosity, the time step for hydrodynamic stability is determined by a generalized Courant condition

$$\Delta t_s = \frac{\delta R}{C_s}\left\{\left[1+\left(C_1+\frac{2C_0^2|\delta U|}{C_s}\right)^2\right]^{\frac{1}{2}}-\left(C_1+\frac{2C_0^2|\delta U|}{C_s}\right)\right\} \quad (172)$$

The time step determined by equation (172) along with the artificial viscosity assure the stability of the hydrodynamic difference equations and the implicit solution of the energy equation guarantees the stability of the thermal conductivity. However, the accuracy of the energy equation suffers if the time step is so large that the thermal front is conducted across more than one zone. The time step for accuracy of thermal conduction is

$$\Delta t_A = \frac{\delta R}{V_{th}} \quad (173)$$

where the velocity of the thermal front is estimated from the similarity solution

$$V_{th} = \frac{\partial T/\partial t}{\partial T/\partial R} \quad (174)$$

Combining equations (173) & (174) and making use of the thermal conductivity term of the energy equation gives

$$\frac{1}{\Delta t_A} = \frac{1}{\delta T}\frac{\partial T}{\partial t}$$

$$\frac{1}{\Delta t_A} = \frac{1}{C\delta T}\frac{\partial}{\partial M}\left[\chi R^{\alpha-1}\frac{\partial T}{\partial R}\right] \quad (175)$$

The corresponding difference equation is

$$\frac{1}{\Delta t_A} = \frac{1}{C_{j-\frac{1}{2}} M_{j-\frac{1}{2}}(T^n_{j-1})} \frac{(R^n_j)^{\alpha-1} X^n_{j-\frac{1}{2}} X^n_{j+\frac{1}{2}} (T^n_{j+\frac{1}{2}} - T^n_{j-\frac{1}{2}})}{X^n_{j-\frac{1}{2}} + X^n_{j+\frac{1}{2}}}$$

$$- \frac{(R^n_{j-1})^{\alpha-1} X^n_{j-3/2} X^n_{j-\frac{1}{2}} (T^n_{j-\frac{1}{2}} - T^n_{j-3/2})}{X^n_{j-3/2} + X^n_{j-\frac{1}{2}}} \qquad (176)$$

3. INPUT VARIABLES AND FORMATS

a. (Problem description in columns 1-60) Format (12A5)

b. NCYCS, NMAT, IALPH, NEDIT, LINMX, INPUT, NELOP, NEPRN, Format (8I10)

NCYCS = maximum number of hydro cycles to be run

NMAT = number of material layers, up to four

IALPH = geometry factor; 1 for slab, 2 for cylindrical and 3 for spherical

NEDIT = cycle interval at which EDITS is called

LINMX = maximum number of lines printed per page 55 is recommended for CDC

INPUT = parameter controlling source of input for cards. INPUT = 0 reads from cards, INPUT = K, input from Kth record on TAPE8.

NELOP = maximum number of electron-ion interaction loops in ELECT for each hydroloop. Usual value is 100-200.

NEPRN = control parameter for printing partial values of electron loop. Usual volume is NELOP-10.

c. NTAPE, NPRIN, IPRIN, JINT, JMIN, JMAX Format (8I10)

NTAPE = Interval for taping time edits of mesh
NTAPE = 0 no time edits taped
NTAPE = -1 only last time edit taped for for restart

NPRIN = Interval for printing and plotting time edits
NPRIN = 0 no time edits printed
NPRIN = -1 only last time edit printed

IPRIN = control parameter for type of time edit print
IPRIN = 0 print and plot
IPRIN = 1 print only
IPRIN = 2 plot only

JINT = interval for plotting and printing mesh points in time edit
JINT = 0 active zones greater than JMIN are plotted

JMIN = minimum mesh point plotted

JMAX = maximum mesh point plotted unles JINT = 0.

d. TSTΦP, DTMIN, TEDIT, REDIT, DTEDT, TMAX, RTMAX Format (8 E10.4)

TSTΦP = maximum problem time

DIMIN = first hydro time step, not used when restarting from tape.

TEDIT = time of first time edit. Additional time edits at times given by

TEDIT = TEDIT * (1 + REDIT) + DTEDT

The run is stopped if the temperature at point RTMAX exceeds TMAX.

e. PΦWW, WLM, TEMP, PRESA Format (8 E10.4)

PΦWW = Laser power in watts/cm^2

WLM = Laser wavelength in micron

TEMP = Ambient temperature in degrees Kelvin

PRESA = Ambient pressure in atmospheres

f. IZT, NZ, THIK, RAT Format (2I10, 6E10.4)(one for each material layer)

IZT = control parameter for type of zoning

IZT = 0, 1 determined by ΔR

IZT = 2 determined by $\Delta V \sim R^{\alpha-1} \Delta R$

NZ = number of zones in layer

THIK = material thickness or radius in centimeters

RAT = ratio for geometric series of changing zone size.

g. MATRL, MMΦD, RHΦ, GAM1, AMWT, VIΦNE, SIGIE Format (2A5, I10, 6E10.4)

MATRL = material name in 10 characters

MMΦD = material model parameter

MMΦD = -1 Gilmore tables for air

= 0 ideal gas

= 1 Hugonoit equation of state

RHΦ = material density at standard conditions

GAM1 = ratio of specific heats of vapor

AMWT = atomic or molecular weight

VIΦNE = ionization energy in ev

SIGIE = slope of electron impact ionization cross section in units πa^2_{Bohr}/ev

h. CHIΦ, BETA, ACL, AEXΦ, VIBΦE, VIBWE, VHNUE, SIGV Format (8E10.4)

CHIϕ = factor in computing temperature dependent thermal conductivity

CHI(J) = CHIϕ(M) * T(J) ** BETA(M)

$$\chi(T) = \chi_o T^{\beta}$$

ACL = linear absorption coefficient at standard density

AEXO = power of density dependence in linear absorption coefficient

$$\alpha(\rho) = \alpha_o \left(\frac{\rho}{\rho_o}\right)^{AEXO}$$

VIBOE = threshold for molecular vibrational excitation by electron impact in ev

VIBWE = bandwidth for molecular vibrational excitation by electron impact in ev.

VANUE = molecular vibrational quanta in ev

SIGV = cross section for molecular vibrational excitation by electron impact at center of band

i. GRUN, HUGC, HUGD, HUGS, ESUB, CVS, TVAP, WFUNE
Format (8E10.4)

GRUN = Grüneisen coefficient for solid

HUGC = C coefficient in Hugonoit equation of state

HUGD = D coefficient in Hugonoit equation of state

HUGS = S coefficient in Hugonoit equation of state

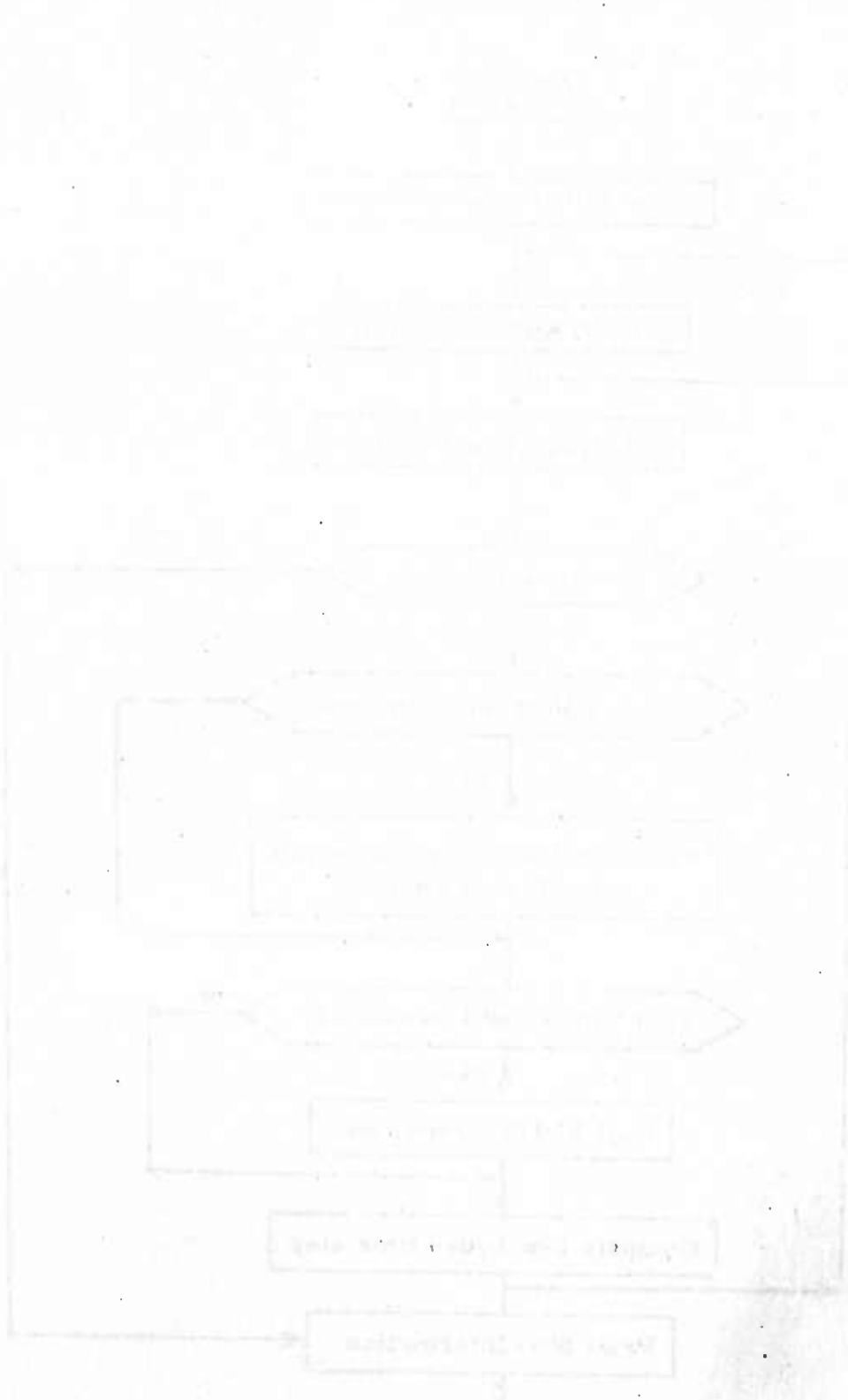
$$PH(J) = C_\mu + D_\mu^2 + S_\mu^3 \text{ in dyne/cm}^2$$

ESUB = sublimation energy of solid in ergs/gm

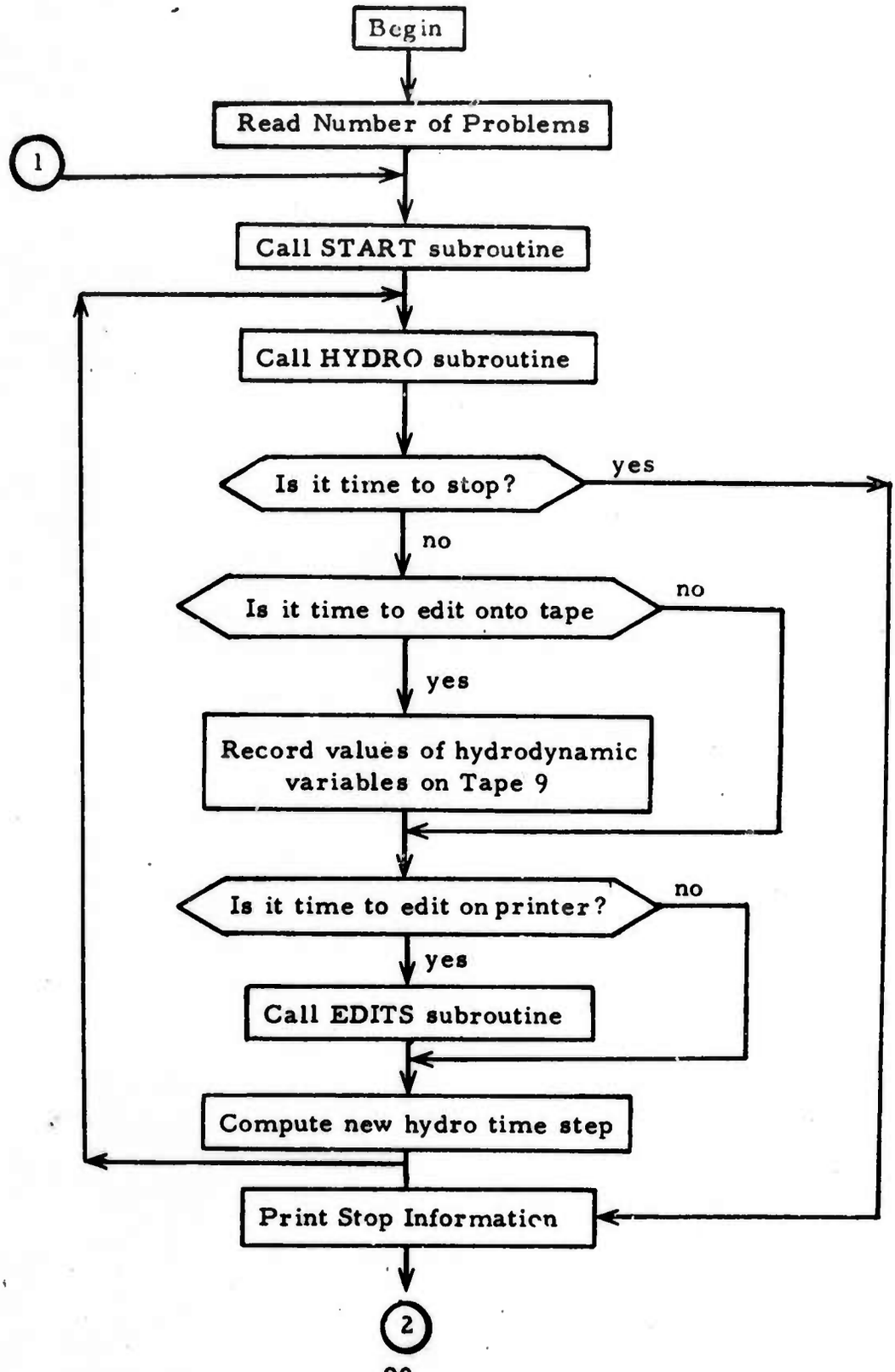
CVS = specific heat of solid in ergs/gm degree

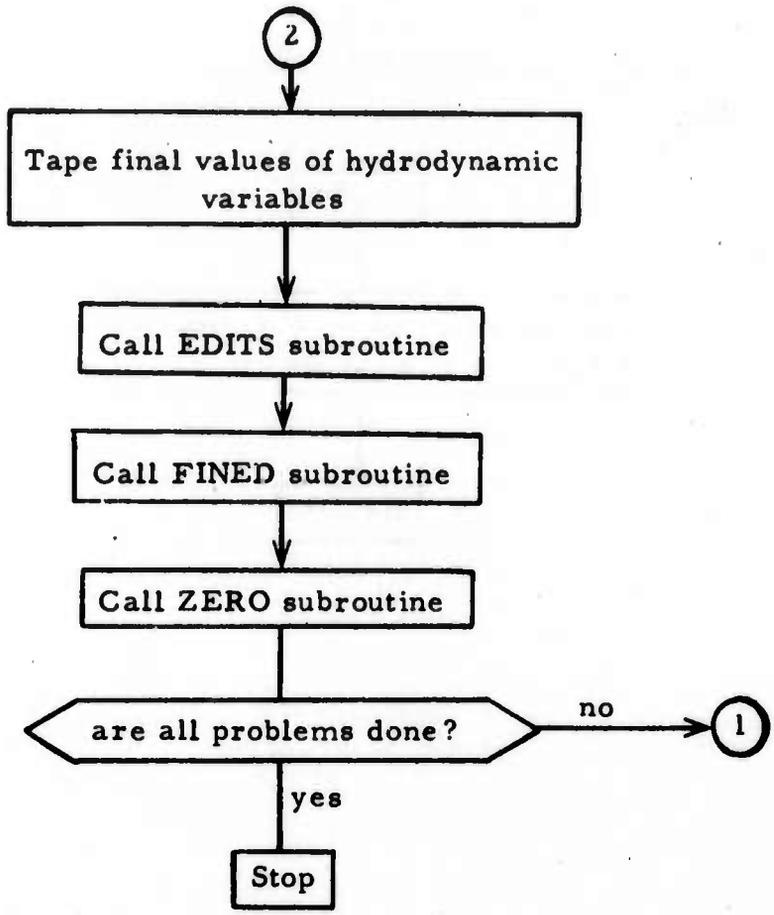
TVAP = vaporization temperature in degrees Kelvin

WFUNE = work function for thermionic emission

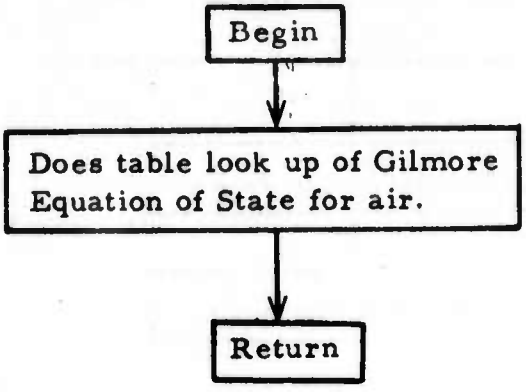


Logical Flow Chart for CELAB Program

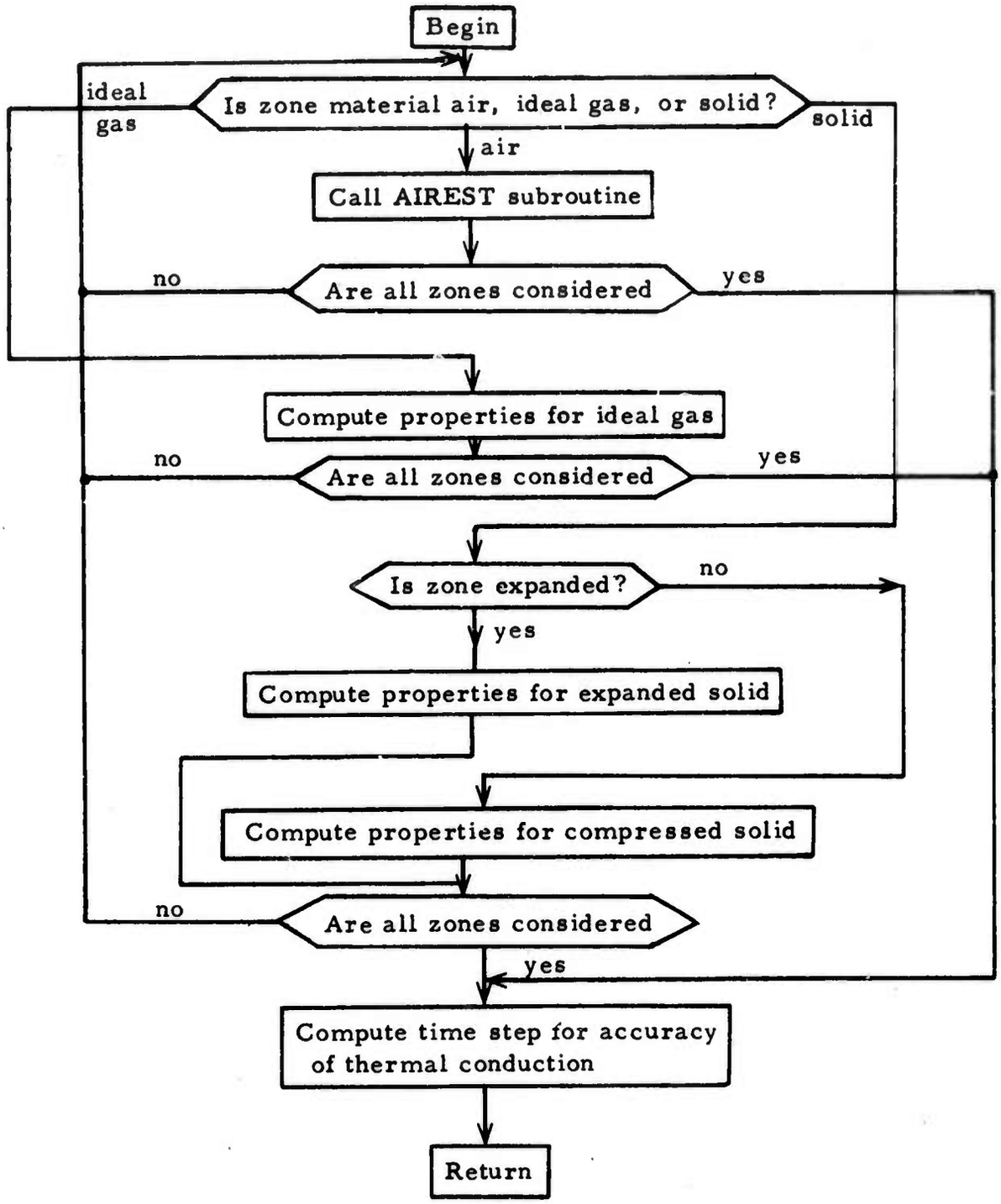



2
Tape final values of hydrodynamic variables
Call EDITS subroutine
Call FINED subroutine
Call ZERO subroutine
are all problems done?
no
1
yes
Stop

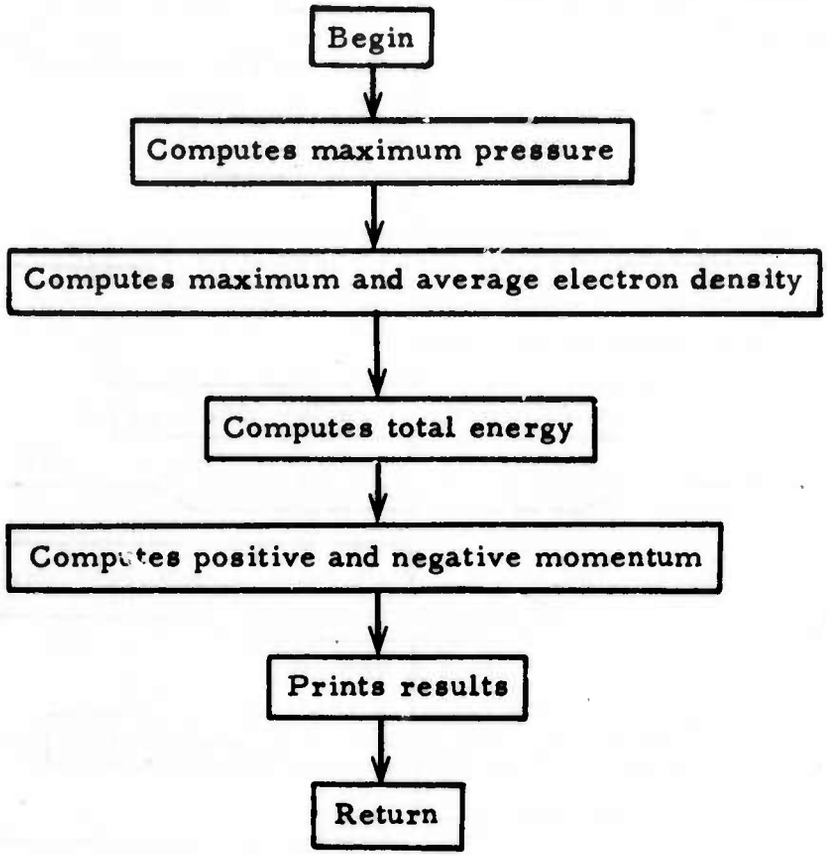
AIREST Subroutine



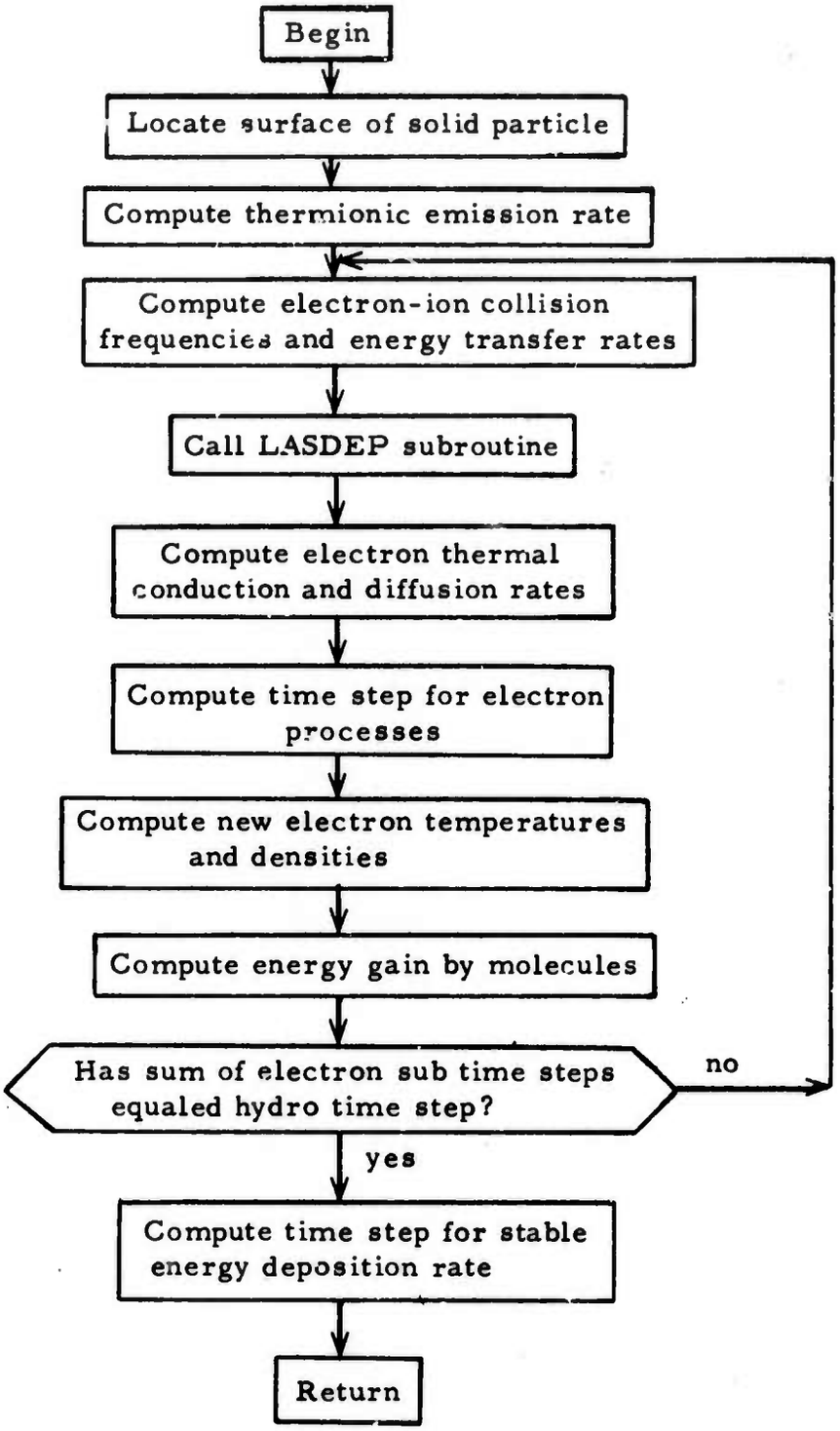
C SOUND Subroutine



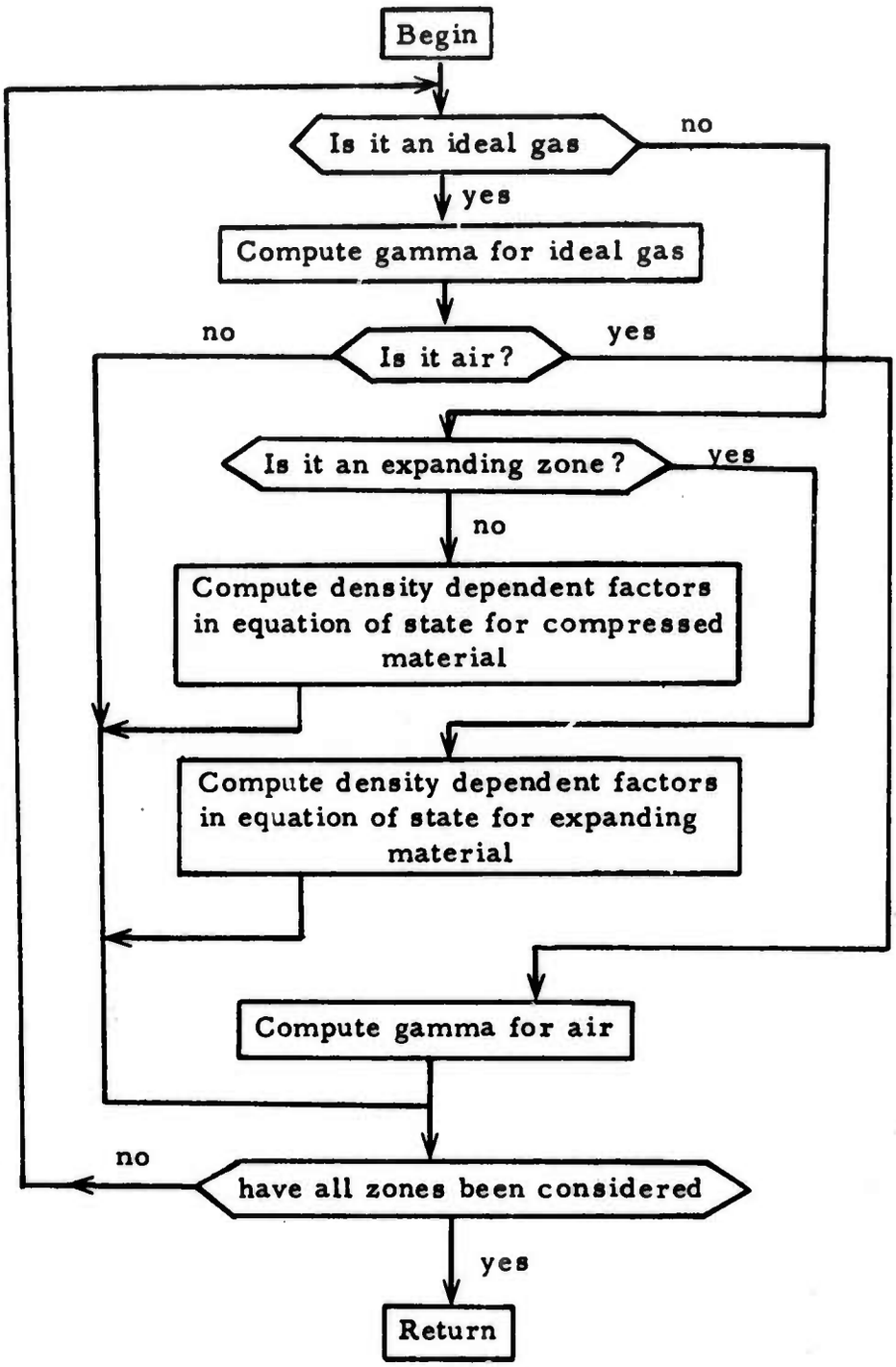
EDITS Subroutine



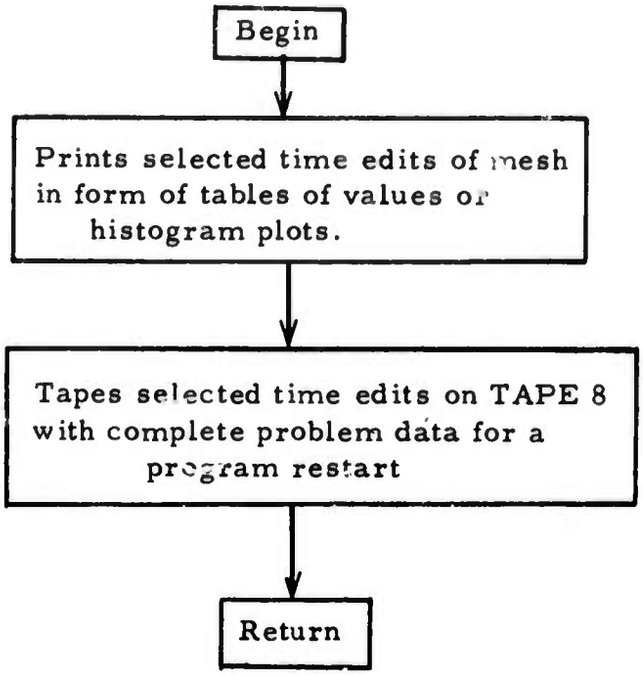
ELECT Subroutine



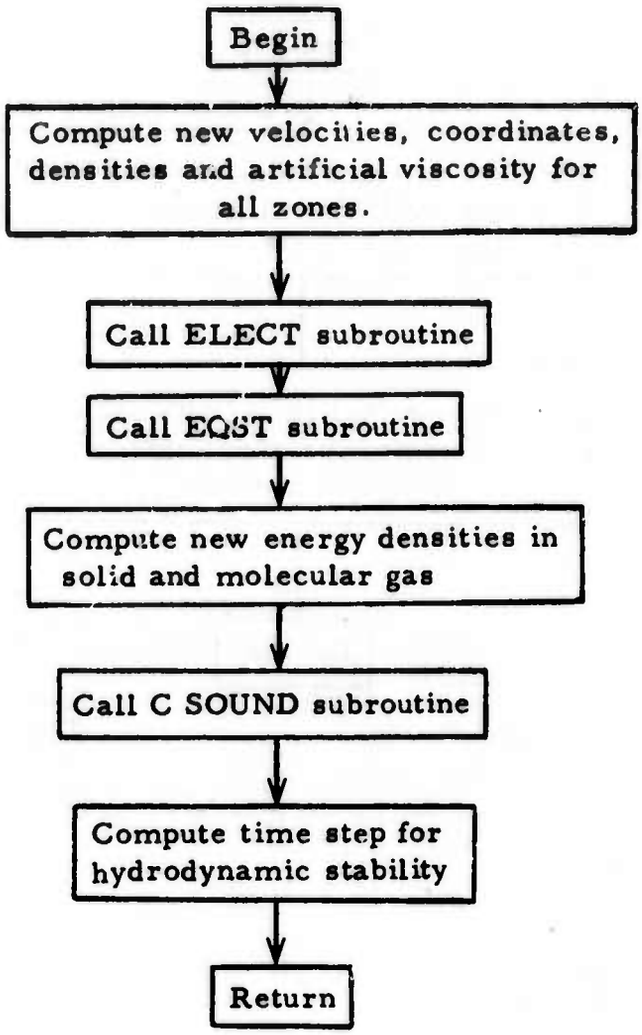
EQST Subroutine



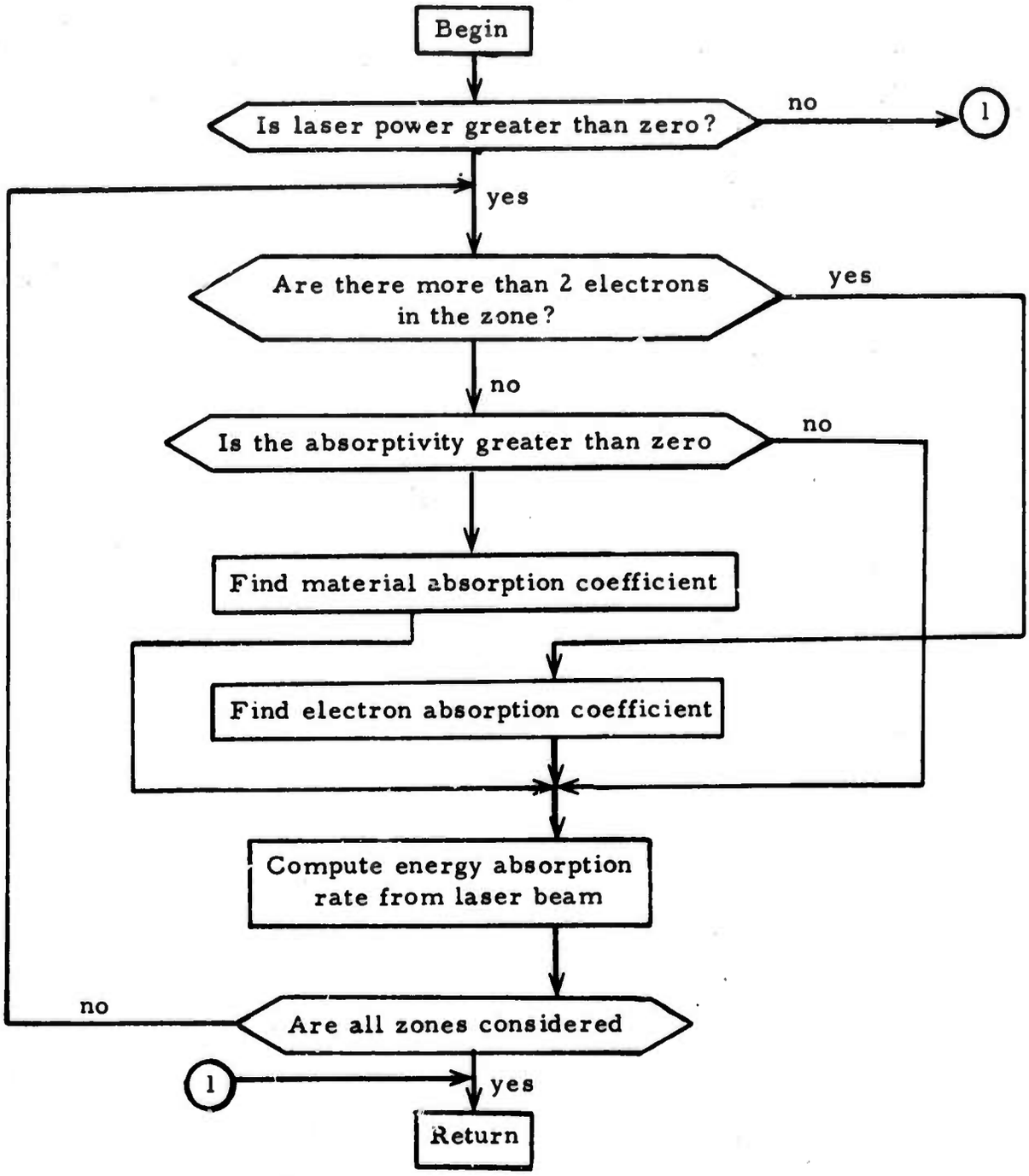
FINED Subroutine



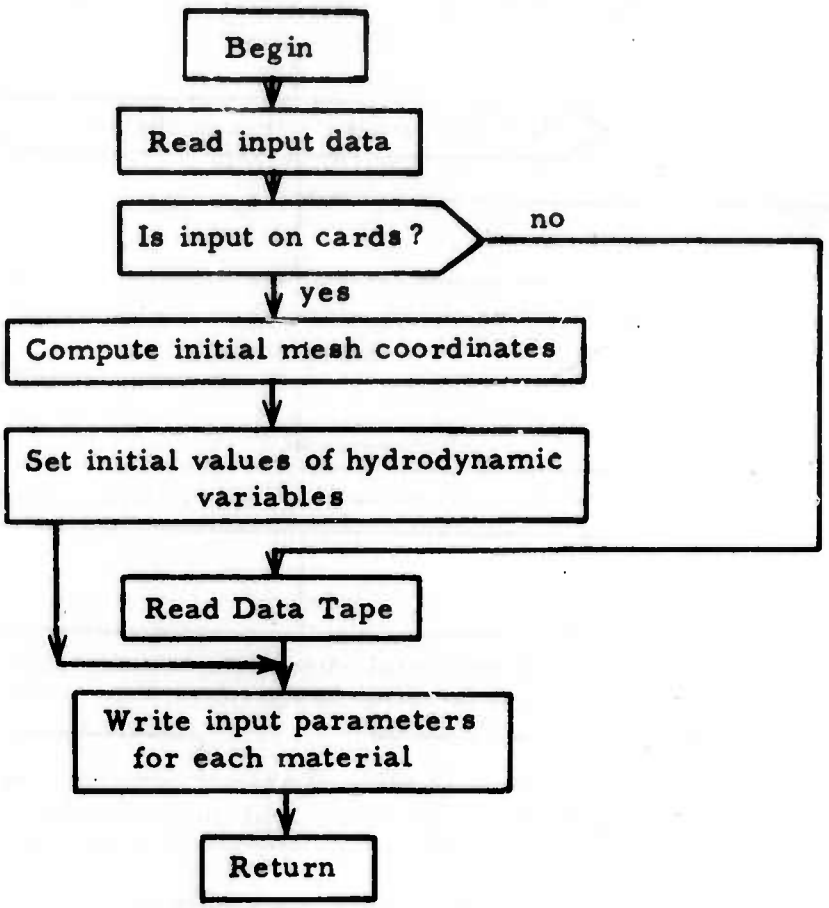
HYDRO Subroutine



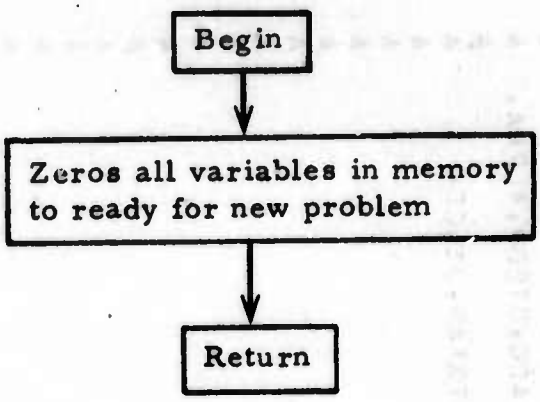
LASDEP Sub-outine



START Subroutine



ZERO Subroutine



```
C                              CELAB MAIN PROGRAM
C
C
C          CONTAMINANT EFFECTS ON LASER AIR BREAKDOWN (CELAB)                  A
C                                                                              A     2
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),          A     3
     .        PE(512),EE(512),TE(512),ANE(512),FM(512),CS(512)                 A     4
      COMMON /NTIME/ NCYCS,N,NELOP,NEL,NEDIT,LINE,LINMX,NTAPE,NPRIN,           A     5
     .        IPRIN,JINT,JMIN,JMAX,NTED,KTAPE,NETP                            A     6
      COMMON /CTIME/ TSTOP,TIME,DTN,DTNH,DTNHS,DTSTAB,DTACC,DTEDP,ALPHA,       A     7
     .        TMAX,TEMP,PRES,EINTI,EUPT,EDP,SOS                               A     8
      COMMON /JPAR/ JOR,JORP,JACM,JAC,JACP,JTMAX,IALPM,JTSTAB,JTACC,           A     9
     .        JTEDP,JSR                                                        A    10
      COMMON /NAME/ IDT(12),MATN(8)                                            A    11
      COMMON /EDTIME/ TEDIT,REDIT,DTEDT                                        A    12
      READ 48, NPROB                                                           A    13
      DO 36 ND00=1,NPROB                                                       A    14
      NETP=0                                                                   A    15
      KTAPE=0                                                                  A    16
      N=0                                                                      A    17
      NEL=0                                                                    A    18
      TIME=0.                                                                  A    19
      NTED=1                                                                   A    20
      NEDCON=0                                                                 A    21
      LINE=0                                                                   A    22
      JDEN=0                                                                   A    23
      CALL START                                                               A    24
      PRINT 44                                                                 A    25
      NED=0                                                                    A    26
      IF (NEDIT) 2,2,1                                                         A    27
    1 NED=1+N/NEDIT                                                            A    28
    2 NED=NED*NEDIT                                                            A    29
      IF (TEDIT-TIME-DTNH) 3,3,4                                               A    30
    3 NEDCON=1                                                                 A    31
      TEDIT=TIME+DTNH                                                          A    32
    4 WRITE (9) TIME,DTNHS,DTNH,N,JORP,(R(J),P(J),Q(J),U(J),D(J),E(J),         A    33
     .        T(J),PE(J),EE(J),TE(J),ANE(J),FM(J),CS(J),J=1,JORP)              A    34
```

```
C     BEGIN TIME LOOP **                                                    A   35
    5 TIME=TIME+DTNH                                                        A   36
      N=N+1                                                                 A   37
      CALL HYDRO (JDEN)                                                     A   38
C     CHECK STOP PARAMETERS **                                              A   39
      IF (NEL .GT. NELOP) GO TO 24                                          A   40
      IF (JDEN) 19,6,20                                                     A   41
    6 IF (NCYCS-N) 7,21,7                                                   A   42
    7 IF (TSTOP-TIME) 22,22,8                                               A   43
    8 IF (TMAX-T(JTMAX)) 23,23,9                                            A   44
    9 IF (NEDCON) 12,12,10                                                  A   45
C     TIME EDIT OF MESH **                                                  A   46
   10 NEDCON=0                                                              A   47
      NTED=NTED+1                                                           A   48
      WRITE (9) TIME,DTNHS,DTNH,N,JAC,(R(J),P(J),Q(J),U(J),D(J),E(J),       A   49
     .       T(J),PE(J),EE(J),TE(J),ANE(J),FM(J),CS(J),J=1,JAC)             A   50
      TEDIT=(1.+REDIT)*TEDIT+DTEDT                                          A   51
      IF (NEDIT) 15,11,11                                                   A   52
   11 CALL EDITS                                                            A   53
      IF (N-NED) 15,14,15                                                   A   54
   12 IF (N-NED) 15,13,15                                                   A   55
   13 CALL EDITS                                                            A   56
   14 NED=NED+NEDIT                                                         A   57
C     COMPUTE NEW TIME STEP **                                              A   58
   15 DTN=DTNH                                                              A   59
      DTNH=0.8*DTSTAB                                                       A   60
      DTNHS=1.26*DTNHS                                                      A   61
      DTNH=AMIN1(DTNHS,DTNH,DTEDP,DTACC)                                    A   62
      DTNHS=DTNH                                                            A   63
      IF (TIME+DTNH-TEDIT) 18,17,16                                         A   64
   16 DTNH=TEDIT-TIME                                                       A   65
   17 NEDCON=1                                                              A   66
   18 DTN=DTN+DTNH                                                          A   67
      GO TO 5                                                               A   68
C     PRINT STOP INFORMATION **                                             A   69
   19 PRINT 39                                                              A   70
```

```
      JDEN=-JDEN                                                          A   71
      PRINT 45, E(JDEN),JDEN,N,TIME                                       A   72
      GO TO 25                                                            A   73
   20 PRINT 39                                                            A   74
      PRINT 42, D(JDEN),JDEN,N,TIME                                       A   75
      GO TO 25                                                            A   76
   21 PRINT 39                                                            A   77
      PRINT 40, N,TIME                                                    A   78
      GO TO 25                                                            A   79
   22 PRINT 39                                                            A   80
      PRINT 41, TIME,N                                                    A   81
      GO TO 25                                                            A   82
   23 PRINT 39                                                            A   83
      PRINT 43, T(JTMAX),JTMAX,TMAX,N,TIME                                A   84
      GO TO 25                                                            A   85
   24 PRINT 39                                                            A   86
      PRINT 47, NEL,NELOP                                                 A   87
   25 IF (NTAPE) 26,28,27                                                 A   88
   26 NETP=NTED+1                                                         A   89
      KTAPE=1                                                             A   90
      GO TO 28                                                            A   91
   27 NETP=1                                                              A   92
      KTAPE=2+(NTED-1)/NTAPE                                              A   93
   28 PRINT 46, KTAPE                                                     A   94
      IF (NEDIT) 33,29,29                                                 A   95
   29 LINE=LINE+6                                                         A   96
      IF (LINMX-LINE) 30,31,32                                            A   97
   30 PRINT 44                                                            A   98
   31 LINE=6                                                              A   99
   32 CALL EDITS                                                          A  100
   33 WRITE (9) TIME,DTNHS,DTNH,N,JAC,(R(J),P(J),Q(J),U(J),D(J),E(J),     A  101
     .       T(J),PF(J),EE(J),TE(J),ANE(J),FM(J),CS(J),J=1,JAC)           A  102
      NTED=NTED+1                                                         A  103
      END FILE 9                                                          A  104
      REWIND 9                                                            A  105
      IF (NPRIN) 35,34,35                                                 A  106
```

```
34 IF (NTAPE) 35,38,35                                                    A  107
35 CALL FINED                                                             A  108
   CALL ZERO                                                              A  109
36 CONTINUE                                                               A  110
   IF (NTAPE) 37,38,37                                                    A  111
37 END FILE 8                                                             A  112
   REWIND 8                                                               A  113
38 STOP                                                                   A  114
39 FORMAT (/,130(1H*),/,130(1H*))                                         A  115
40 FORMAT (4X,21H MAXIMUM CYCLE NUMBER,I5,15H WAS REACHED AT,E10.3,       A  116
  .       4H SEC)                                                         A  117
41 FORMAT (4X,7H TIME =,E10.3,                                            A  118
  .       38H EXCEEDED MAXIMUM TIME AT CYCLE NUMBER,I5)                   A  119
42 FORMAT (4X,10H DENSITY =,E10.3,15H OF ZONE NUMBER,I4,                  A  120
  .       28H IS NEGATIVE AT CYCLE NUMBER,I5,11H AND TIME =,E10.3)        A  121
43 FORMAT (4X,7H TEMP =,E10.3,15H OF ZONE NUMBER,I4,9H EXCEEDS ,          A  122
  .       E10.3,16H AT CYCLE NUMBER,I5,11H AND TIME =,E10.3)              A  123
44 FORMAT (1H1)                                                           A  124
45 FORMAT (4X,9H ENERGY =,E10.3,15H OF ZONE NUMBER,I4,                    A  125
  .       28H IS NEGATIVE AT CYCLE NUMBER,I5,11H AND TIME =,E10.3)        A  126
46 FORMAT (4X,8H KTAPE =,I3,22H TIME EDITS WERE TAPED,/)                  A  127
47 FORMAT (4X,6H NEL =,I4,24H EXCEEDS MAXIMUM NELOP =,I4)                 A  128
48 FORMAT (8I10)                                                          A  129
   END                                                                    A  130
```

```
      SUBROUTINE HYDRO (JDEN)                                                    B
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),            B     2
     .       PE(512),EE(512),TE(512),ANE(512),EM(512),CS(512)                    B     3
      COMMON /RMAS/ ZM(512),ZMHI(512),RNH(512),RA(512),RANH(512)                 B     4
      COMMON /MPROP/ PH(512),GAMD(512),CH1(512),BET(512),CI(512)                 B     5
      COMMON /GJHJ/ GJ(512),HJ(512),DVDT(512)                                    B     6
      COMMON /PMAT/ THIK(4),RAT(4),UV(4),RHO1(4),RHO(4),GRUN(4),HUGC(4),         B     7
     .       HUGD(4),HUGS(4),GAM1(4),ESUB(4),ERAT(4),GAMC(4),AMWT(4),            B     8
     .       CHI0(4),BETA(4),AMU(4),AEX(4),ACL(4),AEXO(4),TOE(4),TVAP(4)         B     9
     .       ,CVS(4),CVG(4),CVSI(4),CVGI(4),VIONE(4),VIONK(4),SIGIE(4),          B    10
     .       SIGIK(4),VIBOE(4),VIBOK(4),VIBWE(4),VIBWK(4),VHNUE(4),              B    11
     .       HNUSQ(4),SIGV(4),SEO(4),SEC(4),WFUNE(4),WFUNK(4),DNM(4)             B    12
      COMMON /MPAR/ NMAT,MMOD(7),JBND(4),IZT(4),NZ(4)                            B    13
      COMMON /JPAR/ JOR,JORP,JACM,JAC,JACP,JTMAX,IALPM,JTSTAB,JTACC,             B    14
     .       JTEDP,JSR                                                           B    15
      COMMON /NTIME/ NCYCS,N,NELOP,NEL,NEDIT,LINE,LINMX,NTAPE,NPRIN,             B    16
     .       IPRIN,JINT,JMIN,JMAX,NTED,KTAPE,NETP                               B    17
      COMMON /CTIME/ TSTOP,TIME,DTN,DTNH,DTNHS,DTSTAB,DTACC,DTEDP,ALPHA,         B    18
     .       TMAX,TEMP,PRES,EINTI,EUPT,EDP,SOS                                  B    19
      DATA COSQH,COSQ2,C1H,C1/2.,.8.,0.125,0.250/                                B    20
C     BEGIN HYDRO LOOP **                                                        B    21
      J2=1                                                                       B    22
      PJP=P(2)+PE(2)+Q(2)                                                        B    23
      RUJP=0.                                                                    B    24
      DO 10 M=1,NMAT                                                             B    25
      J1=J2+1                                                                    B    26
      J2=JBND(M)                                                                 B    27
      DO 10 J=J1,J2                                                              B    28
      PJM=PJP                                                                    B    29
      PJP=P(J+1)+PE(J+1)+Q(J+1)                                                  B    30
      U(J)=U(J)-RA(J)*DTN*ZMHI(J)*(PJP-PJM)                                      B    31
      IF (ABS(U(J))-.01) 1,1,2                                                   B    32
    1 U(J)=0.                                                                    B    33
      GO TO 3                                                                    B    34
    2 RNH(J)=R(J)                                                                B    35
      R(J)=R(J)+DTNH*U(J)                                                        B    36
```

```
      RNH(J)=RNH(J)+R(J)                                                  B  37
      RANH(J)=RA(J)                                                       B  38
      RA(J)=R(J)**IALPM                                                   B  39
      RANH(J)=RANH(J)+RA(J)                                               B  40
    3 RUJM=RUJP                                                           B  41
      RUJP=0.5*RANH(J)*U(J)                                               B  42
      DVDT(J)=(RUJP-RUJM)/ZM(J)                                           B  43
      DOLD=D(J)                                                           B  44
      D(J)=ALPHA*ZM(J)/(RA(J)*R(J)-RA(J-1)*R(J-1))                        B  45
      IF (D(J)) 22,22,4                                                   B  46
C     COMPUTE ARTIFICIAL VISCOSITY **                                     B  47
    4 Q(J)=0.                                                             B  48
      DU=U(J)-U(J-1)                                                      B  49
      DUA=ABS(DU)                                                         B  50
      IF (DUA-1.) 8,5,5                                                   B  51
    5 IF (DU) 7,8,6                                                       B  52
    6 IF (E(J)-ESUB(M)) 7,7,8                                             B  53
    7 Q(J)=-(COSQH*DUA+C1H*CS(J))*(D(J)+DOLD)*DU                          B  54
    8 IF (U(J)) 10,9,10                                                   B  55
    9 IF (J-JAC) 10,11,11                                                 B  56
   10 CONTINUE                                                            B  57
   11 JAC=J                                                               B  58
      JACM=JAC-1                                                          B  59
      JACP=JAC+1                                                          B  60
      CALL ELECT (JDEN)                                                   B  61
      IF (JDEN) 23,12,23                                                  B  62
   12 IF (NEL-NELOP) 13,13,23                                             B  63
   13 CALL EQST                                                           B  64
C     BEGIN THERMAL CONDUCTIVITY LOOP **                                  B  65
      AJP=0.                                                              B  66
      CJM=0.                                                              B  67
      ZTP=0.                                                              B  68
      YJP=RNH(2)/CHI(2)                                                   B  69
      BYTP=BET(2)*YJP/T(2)                                                B  70
      DO 14 J=2,JAC                                                       B  71
      DVH=0.5*DVDT(J)*DTNH                                                B  72
```

```
      AJ=AJP/ZM(J)                                                              B   73
      YJM=YJP                                                                   B   74
      ZTM=ZTP                                                                   B   75
      BYTM=BYTP                                                                 B   76
      YJP=(RNH(J+1)-RNH(J))/CHI(J+1)                                            B   77
      BYTP=BET(J+1)*YJP/T(J+1)                                                  B   78
      YJPM=YJM+YJP                                                              B   79
      TOY=(T(J+1)-T(J))/YJPM                                                    B   80
      ZJ=RANH(J)*DTNH/YJPM                                                      B   81
      ZTP=2.*ZJ*(T(J+1)-T(J))                                                   B   82
      AJP=(1.-TOY*BYTM)*ZJ*CI(J)                                                B   83
      BJ=1.+GAMD(J)*DVH*(AJP+CJM)/ZM(J)                                         B   84
      CJM=(1.+TOY*BYTP)*ZJ*CI(J+1)                                              B   85
      CJ=CJM/ZM(J)                                                              B   86
      DJ=FM(J)-(GAMD(J)*E(J)+PH(J)+P(J)+Q(J)+Q(J))*DVH*(ZTP-ZTM)/ZM(J)          B   87
      DEN=BJ-AJ*GJ(J-1)                                                         B   88
      GJ(J)=CJ/DEN                                                              B   89
   14 HJ(J)=(DJ+AJ*HJ(J-1))/DEN                                                 B   90
      WJ=0.                                                                     B   91
      DO 15 I=1,JACM                                                            B   92
      J=JACP-I                                                                  B   93
      WJ=GJ(J)*WJ+HJ(J)                                                         B   94
      E(J)=E(J)+WJ                                                              B   95
      IF (E(J)) 21,21,15                                                        B   96
   15 P(J)=PH(J)+GAMD(J)*E(J)                                                   B   97
      CALL CSOUND                                                               B   98
C     COMPUTE STABLE TIME STEP **                                               B   99
      DTSTAB=0.                                                                 B  100
      DO 20 J=2,JAC                                                             B  101
      IF (Q(J)) 17,17,16                                                        B  102
   16 DU=C1+COSQ2*ABS(U(J)-U(J-1))/CS(J)                                        B  103
      DTI=CS(J)/((R(J)-R(J-1))*(SQRT(1.+DU*DU)-DU))                             B  104
      GO TO 18                                                                  B  105
   17 DTI=CS(J)/(R(J)-R(J-1))                                                   B  106
   18 IF (DTSTAB-DTI) 19,19,20                                                  B  107
   19 DTSTAB=DTI                                                                B  108
```

```
      JTSTAB=J                                                    B  109
   20 CONTINUE                                                    B  110
      DTSTAB=1./DTSTAB                                            B  111
      RETURN                                                      B  112
   21 JDEN=-J                                                     B  113
      RETURN                                                      B  114
   22 JDEN=J                                                      B  115
   23 RETURN                                                      B  116
      END                                                         B  117
```

```
      SUBROUTINE ELECT (JDEN)                                                C
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),        C     2
     .      PE(512),EE(512),TE(512),ANE(512),FM(512),CS(512)                 C     3
      COMMON /RMAS/ ZM(512),ZMHI(512),RNH(512),RA(512),RANH(512)             C     4
      COMMON /GJHJ/ DNE(512),DEE(512),DVDT(512)                              C     5
      COMMON /FREQ/ CNU(512),FE(512)                                         C     6
      COMMON /PMAT/ THIK(4),RAT(4),DV(4),RHO1(4),RHO(4),GRUN(4),HUGC(4),     C     7
     .      HUGD(4),HUGS(4),GAM1(4),ESUB(4),ERAT(4),GAMC(4),AMWT(4),         C     8
     .      CHI0(4),BETA(4),AMU(4),AEX(4),ACL(4),AEXO(4),TOE(4),TVAP(4)      C     9
     .      ,CVS(4),CVG(4),CVSI(4),CVGI(4),VIONE(4),VIONK(4),SIGIE(4),       C    10
     .      SIGIK(4),VIBOE(4),VIBOK(4),VIBWE(4),VIBWK(4),VHNUE(4),           C    11
     .      HNUSQ(4),SIGV(4),SEO(4),SEC(4),WFUNE(4),WFUNK(4),DNM(4)          C    12
      COMMON /MPAR/ NMAT,MMOD(7),JBND(4),IZT(4),NZ(4)                        C    13
      COMMON /JPAR/ JOR,JORP,JACM,JAC,JACP,JTMAX,IALPM,JTSTAB,JTACC,         C    14
     .      JTEDP,JSR                                                        C    15
      COMMON /NTIME/ NCYCS,N,NELOP,NEL,NEDIT,LINE,LINMX,NTAPE,NPRIN,         C    16
     .      IPRIN,JINT,JMIN,JMAX,NTED,KTAPE,NETP                            C    17
      COMMON /CTIME/ TSTOP,TIME,DTN,DTNH,DTNHS,DTSTAB,DTACC,DTEDP,ALPHA,     C    18
     .      TMAX,TEMP,PRES,EINTI,EDPT,EDP,SOS                                C    19
      COMMON /EPAR/ ACPAO,REC,SIGO,SIGC,DIFI,ATIE,ALO,AKBT,AKB,GAME,         C    20
     .      FNEL,NEPRN                                                       C    21
      ICON=0                                                                 C    22
      NEL=0                                                                  C    23
      DTFN=DTNH*FNEL                                                         C    24
      DTFNT=DTFN+DTFN                                                        C    25
      DTFNO=10.*DTFNT                                                        C    26
      ETIME=TIME-DTNH                                                        C    27
      DO 1 J=2,JAC                                                           C    28
      CNU(J)=0.                                                              C    29
      FE(J)=0.                                                               C    30
    1 FM(J)=0.                                                               C    31
C     DETERMINE SURFACE OF SOLID PARTICLE **                                 C    32
      J2=1                                                                   C    33
      DO 2 MS=1,NMAT                                                         C    34
      J1=J2+1                                                                C    35
      J2=JBND(MS)                                                            C    36
```

```
      DO 2 J=J1,J2                                                    C   37
      IF (E(J)-ESUB(MS)) 2,2,3                                        C   38
    2 TE(J)=T(J)                                                      C   39
    3 JS=J-1                                                          C   40
      DO 4 ME=1,NMAT                                                  C   41
      IF (JBND(ME)-JS) 4,5,5                                          C   42
    4 CONTINUE                                                        C   43
    5 JV=JS+1                                                         C   44
      IF (JS-JSR) 6,7,7                                               C   45
    6 JSR=JS                                                          C   46
      TE(JV)=T(JV)                                                    C   47
      ANE(JV)=D(JV)*DNM(MS)*EXP(-0.5*VIONK(MS)/T(JV))                 C   48
      EE(JV)=AKBT*ANE(JV)*TE(JV)/D(JV)                                C   49
C     THERMIONIC EMISSION **                                          C   50
    7 SOS=T(JS)**2*EXP(ATIE-WFUNK(ME)/T(JS))/(R(JV)-R(JS))            C   51
      EOS=AKBT*T(JS)*SOS/D(JV)                                        C   52
      DO 8 J=JV,JAC                                                   C   53
      IF (ANE(J)-1.0) 9,9,8                                           C   54
    8 CONTINUE                                                        C   55
    9 JEL=J-1                                                         C   56
      JELP=JEL+1                                                      C   57
      IF (JEL-JS) 56,10,12                                            C   58
   10 ANE(JV)=ANE(JV)+SOS*DTNH                                        C   59
      EE(JV)=EE(JV)+EOS*DTNH                                          C   60
      TE(JV)=T(JS)                                                    C   61
      CNU(JV)=ACPAO*SQRT(TE(JV))*DNM(MS)*D(JV)*SIGO                   C   62
      CALL LASDEP                                                     C   63
      DO 11 J=2,JAC                                                   C   64
   11 FM(J)=FE(J)*DTNH                                                C   65
      EDPT=EDPT+EDP*DTNH                                              C   66
      GO TO 51                                                        C   67
C     ELECTRON-ION INTERACTION LOOP **                                C   68
   12 NEL=NEL+1                                                       C   69
      J2=JS                                                           C   70
      DTTE=1.                                                         C   71
      DO 21 M=MS,NMAT                                                 C   72
```

```
      J1=J2+1                                                          C   73
      J2=MIN0(JBND(M),JEL)                                             C   74
      DO 21 J=J1,J2                                                    C   75
      IDO=0                                                            C   76
      ANN=DNM(M)*D(J)                                                  C   77
      TEJ=TE(J)                                                        C   78
      IF (EE(J)) 14,14,13                                              C   79
   13 TH=SQRT(TEJ)                                                     C   80
      FFAC=ACPA0*TH                                                    C   81
      CFI=SIGIK(M)*(VIONK(M)+TEJ+TEJ)                                  C   82
      AA=EXP(-VIONK(M)/TEJ)                                            C   83
      DNR=FFAC*CFI                                                     C   84
      DNI=DNR*AA*ANN                                                   C   85
      DNR=DNR*ANE(J)*REC*ANE(J)/(TH*TEJ)                               C   86
      DNE(J)=ANE(J)*(DNI-DNR)                                          C   87
      CFI=CFI*AA                                                       C   88
      DEI=AKB*VIONK(M)*DNE(J)/D(J)                                     C   89
      AA=VIBOK(M)/TEJ                                                  C   90
      BB=VIBWK(M)/TEJ                                                  C   91
      VFAC=EXP(-AA)*(1.+AA-(1.+AA+BB)*EXP(-BB))                        C   92
      CFV=SIGV(M)*VFAC                                                 C   93
      DEV=HNUSQ(M)*CFV/(TEJ*T(J))                                      C   94
      AL=(ALO+ALOG(TEJ**3/ANE(J)))/TEJ                                 C   95
      CFC=SIGC*AL**2                                                   C   96
      DEC=SEC(M)*AL/TEJ                                                C   97
      CNU(J)=FFAC*(ANE(J)*CFC+ANN*(SIGO+CFV+CFI))                      C   98
      FE(J)=FFAC*AKB*(TEJ-T(J))*ANE(J)*(ANE(J)*DEC+ANN*(SEO(M)+DEV))/  C   99
     *      D(J)                                                       C  100
      DEE(J)=-FE(J)-DEI                                                C  101
      DKT=DEE(J)/EE(J)-DNI+DNR                                         C  102
      DTT=ABS(0.1/DKT)                                                 C  103
      IF (DTT-DTFNT) 14,14,15                                          C  104
   14 TE(J)=T(J)                                                       C  105
      AA=FE(J)                                                         C  106
      EE(J)=AKBT*ANE(J)*TE(J)/D(J)                                     C  107
      FE(J)=0.                                                         C  108
```

```
      FM(J)=FM(J)+AA-EE(J)                                              C  109
      TH=SQRT(TE(J))                                                    C  110
      FFAC=ACPAO*TH                                                     C  111
      CFI=SIGIK(M)*(VIONK(M)+TE(J)+TE(J))                               C  112
      AA=EXP(-VIONK(M)/TE(J))                                           C  113
      DNR=FFAC*CFI                                                      C  114
      DNI=DNR*AA*ANN                                                    C  115
      DNR=DNR*ANE(J)*REC*ANE(J)/(TH*TE(J))                              C  116
      DNE(J)=ANE(J)*(DNI-DNR)                                           C  117
      DEE(J)=AKB*VIONK(M)*DNE(J)/D(J)                                   C  118
      CFI=CFI*AA                                                        C  119
      AA=VIBOK(M)/TE(J)                                                 C  120
      BB=VIBWK(M)/TE(J)                                                 C  121
      VFAC=EXP(-AA)*(1.+AA-(1.+AA+BB)*EXP(-BB))                         C  122
      CFV=SIGV(M)*VFAC                                                  C  123
      DEV=HNUSQ(M)*CFV/(TE(J)*T(J))                                     C  124
      AL=(ALO+ALOG(TE(J)**3/ANE(J)))/TE(J)                              C  125
      CFC=SIGC*AL**2                                                    C  126
      DEC=SEC(M)*AL/TE(J)                                               C  127
      CNU(J)=FFAC*(ANE(J)*CFC+ANN*(SIGO+CFV+CFI))                       C  128
      GO TO 21                                                          C  129
   15 IF (DTT-DTNFO) 16,16,20                                           C  130
   16 IF (IDO) 17,17,20                                                 C  131
   17 IDO=1                                                             C  132
      IF (DKT) 18,21,19                                                 C  133
   18 TEJ=0.95*TEJ                                                      C  134
      GO TO 13                                                          C  135
   19 TEJ=1.05*TEJ                                                      C  136
      GO TO 13                                                          C  137
   20 DTTE=AMIN1(DTT,DTTE)                                              C  138
   21 CONTINUE                                                          C  139
      DNE(JS+1)=DNE(JS+1)+SOS                                           C  140
      DEE(JS+1)=DEE(JS+1)+EOS                                           C  141
      CNU(JELP)=ACPAO*SQRT(TE(JELP))*DNM(NMAT)*D(JELP)*SIGO             C  142
      CALL LASDEP                                                       C  143
C     BEGIN DIFFUSION LOOP **                                           C  144
```

```
      J1=JS+1                                                          C  145
      DTDIF=0.                                                         C  146
      DTCON=0.                                                         C  147
      DYJP=DIFI*CNU(J1)*(RNH(J1)-RNH(J1-1))/TE(J1)                     C  148
      CYJP=AKBT*ANE(J1)/DYJP                                           C  149
      IF (CYJP) 22,23,22                                               C  150
   22 CYJP=1./CYJP                                                     C  151
   23 DNEJP=0.                                                         C  152
      CEEJP=0.                                                         C  153
      DO 30 J=J1,JEL                                                   C  154
      DYJM=DYJP                                                        C  155
      CYJM=CYJP                                                        C  156
      DNEJM=DNEJP                                                      C  157
      CEEJM=CEEJP                                                      C  158
      DYJP=DIFI*CNU(J+1)*(RNH(J+1)-RNH(J))/TE(J)                       C  159
      DNEJP=RANH(J)/(DYJM+DYJP)                                        C  160
      IF (DTDIF-DNEJP*ZMHI(J)) 24,25,25                                C  161
   24 DTDIF=DNEJP*ZMHI(J)                                              C  162
   25 CEEJP=DNEJP*(EE(J+1)-EE(J))                                      C  163
      DNEJP=DNEJP*(ANE(J+1)-ANE(J))                                    C  164
      DNE(J)=DNE(J)+D(J)*(DNEJP-DNEJM)/ZM(J)                           C  165
      CYJP=AKBT*ANE(J+1)/DYJP                                          C  166
      IF (CYJP) 26,29,26                                               C  167
   26 CYJP=1./CYJP                                                     C  168
      CEEJP=RANH(J)/(CYJM+CYJP)                                        C  169
      IF (DTCON-CEEJP*ZMHI(J)) 27,28,28                                C  170
   27 DTCON=CEEJP*ZMHI(J)                                              C  171
   28 CEEJP=CEEJP*(TE(J+1)-TE(J))                                      C  172
      GO TO 30                                                         C  173
   29 CEEJP=0.                                                         C  174
   30 DEE(J)=DEE(J)+(CEEJP-CEEJM)/ZM(J)                                C  175
      IF (DNEJP) 31,32,32                                              C  176
   31 DNE(JELP)=-D(J)*DNEJP/ZM(J)                                      C  177
   32 IF (CEEJP) 33,34,34                                              C  178
   33 DEE(JELP)=-CEEJP/ZM(J)                                           C  179
   34 DTEE=1.                                                          C  180
```

```
      DTNE=1.                                                          C  181
      DO 40 J=J1,JEL                                                   C  182
      IF (DEE(J)) 35,37,37                                             C  183
   35 DTEST=-EE(J)/DEE(J)                                              C  184
      IF (DTEST-DTEE) 36,37,37                                         C  185
   36 DTEE=DTEST                                                       C  186
   37 IF (DNE(J)) 38,40,40                                             C  187
   38 DTEST=-ANE(J)/DNE(J)                                             C  188
      IF (DTEST-DTNE) 39,40,40                                         C  189
   39 DTNE=DTEST                                                       C  190
   40 CONTINUE                                                         C  191
      DTEL=AMAX1(2.*DTTE,DTFM)                                         C  192
      DTEL=0.5*AMIN1(DTEL,DTNE,DTEE,DTNH)                              C  193
      IF (TIME-ETIME-DTEL) 41,42,43                                    C  194
   41 DTEL=TIME-ETIME                                                  C  195
   42 ICON=1                                                           C  196
   43 ETIME=ETIME+DTEL                                                 C  197
      DO 46 J=J1,JELP                                                  C  198
      ANE(J)=ANE(J)+DNE(J)*DTEL                                        C  199
      IF (ANE(J)) 56,46,44                                             C  200
   44 EE(J)=EE(J)+DEE(J)*DTEL                                          C  201
      IF (EE(J)) 57,46,45                                              C  202
   45 AA=D(J)*EE(J)                                                    C  203
      TE(J)=0.5*(TE(J)+AA/(AKBT*ANE(J)))                               C  204
      PE(J)=GAME*AA                                                    C  205
   46 CONTINUE                                                         C  206
      EDPT=EDPT+EDP*DTEL                                               C  207
      DO 47 J=2,JAC                                                    C  208
   47 FM(J)=FM(J)+FE(J)*DTEL                                           C  209
      IF (ICON) 51,48,51                                               C  210
   48 IF (NEL-NELOP) 49,49,58                                          C  211
   49 DO 50 J=2,JS                                                     C  212
   50 FE(J)=0.                                                         C  213
      GO TO 12                                                         C  214
   51 DTEDP=0.                                                         C  215
      J2=1                                                             C  216
```

```
      DO 53 M=1,NMAT                                                    C  217
      J1=J2+1                                                           C  218
      J2=MIN0(JBND(M),JAC)                                              C  219
      DO 53 J=J1,J2                                                     C  220
      DTI=FM(J)/AMAX1(E(J),ESUB(M))                                     C  221
      IF (DTEDP-DTI) 52,53,53                                           C  222
   52 DTEDP=DTI                                                         C  223
      JTEDP=J                                                           C  224
   53 CONTINUE                                                          C  225
      DTEDP=DTEDP/DTNH                                                  C  226
      IF (DTEDP-1.) 54,54,55                                            C  227
   54 DTEDP=1.                                                          C  228
      RETURN                                                            C  229
   55 DTEDP=1./DTEDP                                                    C  230
      RETURN                                                            C  231
   56 JDEN=J                                                            C  232
      RETURN                                                            C  233
   57 JDEN=-J                                                           C  234
   58 RETURN                                                            C  235
      END                                                               C  236
```

```
      SUBROUTINE LASDEP                                                 D
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),   D     2
     .      PE(512),EE(512),TE(512),ANE(512),EM(512),CS(512)           D     3
      COMMON /RMAS/ ZM(512),ZMHI(512),RNH(512),RA(512),RANH(512)        D     4
      COMMON /GJHJ/ DEE(512),DNE(512),DVDT(512)                         D     5
      COMMON /FREQ/ CNU(512),FE(512)                                    D     6
      COMMON /PMAT/ THIK(4),RAT(4),DV(4),RHO1(4),RHO(4),GRUN(4),HUGC(4),D     7
     .      HUGD(4),HUGS(4),GAM1(4),ESUB(4),ERAT(4),GAMC(4),AMWT(4),    D     8
     .      CHI0(4),BETA(4),AMU(4),AEX(4),ACL(4),AEXO(4),TOE(4),TVAP(4) D     9
     .      ,CVS(4),CVG(4),CVSI(4),CVGI(4),VIONE(4),VIONK(4),SIGIE(4),  D    10
     .      SIGIK(4),VIBOE(4),VIBOK(4),VIBWE(4),VIBWK(4),VHNUE(4),      D    11
     .      HNUSQ(4),SIGV(4),SEO(4),SEC(4),WFUNE(4),WFUNK(4),DNM(4)     D    12
      COMMON /MPAR/ NMAT,MMOD(7),JBND(4),IZT(4),NZ(4)                   D    13
      COMMON /JPAR/ JOR,JORP,JACM,JAC,JACP,JTMAX,IALPM,JTSTAB,JTACC,    D    14
     .      JTEDP,JSR                                                   D    15
      COMMON /CTIME/ TSTOP,TIME,DTN,DTNH,DTNHS,DTSTAB,DTACC,DTEDP,ALPHA,D    16
     .      TMAX,TEMP,PRES,EINTI,EUPT,EDP,SOS                           D    17
      COMMON /LASER/ POW,WL,WF,CRITE,EC                                 D    18
      DATA ZX4,ZEXP,C2,C6,C24/0.020,88.72,0.5,0.166666667,0.0416666667/ D    19
      DTEDP=0.                                                          D    20
      EDP=0.                                                            D    21
      P=POW                                                             D    22
      M=NMAT                                                            D    23
      IF (POW) 14,14,1                                                  D    24
C     BEGIN ENERGY DEPOSITION LOOP **                                   D    25
    1 I2=JAC-JBND(M-1)                                                  D    26
      I1=MAX0(1,JACP-JBND(M))                                           D    27
      DO 12 I=I1,I2                                                     D    28
      J=JACP-I                                                          D    29
      IF (ANE(J)-2.) 2,4,4                                              D    30
    2 IF (AMU(M)) 11,11,3                                               D    31
    3 A=AMU(M)*D(J)**AEX(M)                                             D    32
      AE=0.                                                             D    33
      GO TO 6                                                           D    34
    4 A=ANE(J)*CNU(J)*EC                                                D    35
      AE=A                                                              D    36
```

```
      IF (AMU(M)) 6,6,5                                              D   37
    5 A=A+AMU(M)*D(J)**AEX(M)                                        D   38
    6 DR=R(J)-R(J-1)                                                 D   39
      ADR=A*DR                                                       D   40
      IF (ADR-ZX4) 7,8,8                                             D   41
    7 EXM=1.-(C2-(C6-C24*ADR)*ADR)*ADR                               D   42
      FET=PJ*RA(J)*ADR*EXM                                           D   43
      EXM=1.-EXM*ADR                                                 D   44
      GO TO 10                                                       D   45
    8 IF (ADR-ZEXP) 9,13,13                                          D   46
    9 EXM=EXP(-ADR)                                                  D   47
      FET=PJ*RA(J)*(1.-EXM)                                          D   48
   10 PJ=PJ*EXM                                                      D   49
      EDP=EDP+FET                                                    D   50
      FET=FET/ZM(J)                                                  D   51
      DEEL=FET*AE/A                                                  D   52
      FE(J)=FE(J)+FET-DEEL                                           D   53
      DEE(J)=DEE(J)+DEEL                                             D   54
      IF (PJ) 14,14,12                                               D   55
   11 FE(J)=0.                                                       D   56
   12 CONTINUE                                                       D   57
      M=M-1                                                          D   58
C     BRANCH FOR HIGH DEPOSITION RATE **                             D   59
      IF (M) 14,14,1                                                 D   60
   13 FET=PJ*RA(J)                                                   D   61
      EDP=EDP+FET                                                    D   62
      FET=FET/ZM(J)                                                  D   63
      DEEL=FET*AE/A                                                  D   64
      FE(J)=FE(J)+FET-DEEL                                           D   65
      DEE(J)=DEE(J)+DEEL                                             D   66
   14 RETURN                                                         D   67
      END                                                            D   68
```

```
      SUBROUTINE EQST                                                    E     1
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),    E     2
     .       PE(512),EE(512),TE(512),ANE(512),FM(512),CS(512)            E     3
      COMMON /MPROP/ PH(512),GAMD(512),CHI(512),BET(512),CI(512)         E     4
      COMMON /EQSF/ EXFAC(512),SQFAC(512)                                E     5
      COMMON /PMAT/ THIK(4),RAT(4),DV(4),RHO1(4),RHO(4),GRUN(4),HUGC(4), E     6
     .       HUGD(4),HUGS(4),GAM1(4),ESUB(4),ERAT(4),GAMC(4),AMWT(4),    E     7
     .       CHIO(4),BETA(4),AMU(4),AEX(4),ACL(4),AEXO(4),TOE(4),TVAP(4) E     8
     .       ,CVS(4),CVG(4),CVSI(4),CVGI(4),VIONE(4),VIONK(4),SIGIE(4),  E     9
     .       SIGIK(4),VIBOE(4),VIBOK(4),VIBWE(4),VIBWK(4),VHNUE(4),      E    10
     .       HNUSQ(4),SIGV(4),SEO(4),SEC(4),WFUNE(4),WFUNK(4),DNM(4)     E    11
      COMMON /MPAR/ NMAT,MMOD(7),JBND(4),IZT(4),NZ(4)                    E    12
      COMMON /JPAR/ JOR,JORP,JACM,JAC,JACP,JTMAX,IALPM,JTSTAB,JTACC,     E    13
     .       JTEDP,JSR                                                   E    14
      M=0                                                                E    15
      J2=1                                                               E    16
    1 M=M+1                                                              E    17
      J1=J2+1                                                            E    18
      J2=MIN0(JAC,JBND(M))                                               E    19
      IF (MMOD(M)) 8,2,4                                                 E    20
C     IDEAL GAS EQUATION OF STATE **                                     E    21
    2 DO 3 J=J1,J2                                                       E    22
    3 GAMD(J)=GAM1(M)*D(J)                                               E    23
      IF (M-NMAT) 1,10,10                                                E    24
C     HUGONOIT EQUATION OF STATE **                                      E    25
    4 DO 7 J=J1,J2                                                       E    26
      EMU=D(J)/RHO(M)-1.                                                 E    27
      IF (EMU) 5,6,6                                                     E    28
C     FOR EXPANDED ZONES **                                              E    29
    5 V1=RHO(M)/D(J)                                                     E    30
      EXFAC(J)=ESUB(M)*EXP(ERAT(M)*(1.-V1)*V1)                           E    31
      PH(J)=ESUB(M)-EXFAC(J)                                             E    32
      SQFAC(J)=(GRUN(M)-GAM1(M))*SQRT(D(J)/RHO(M))                       E    33
      GAMD(J)=(GAM1(M)+SQFAC(J))*D(J)                                    E    34
      PH(J)=-GAMD(J)*PH(J)                                               E    35
      GO TO 7                                                            E    36
```

```
C       FOR COMPRESSED ZONES **                                          E   37
      6 PH(J)=((HUGS(M)*EMU+HUGD(M))*EMU+HUGC(M))*EMU                    E   38
        PH(J)=PH(J)*(1.-0.5*GRUN(M)*EMU*RHO(M)/D(J))                     E   39
        GAMD(J)=GRUN(M)*D(J)                                             E   40
      7 CONTINUE                                                         E   41
        IF (M-NMAT) 1,10,10                                              E   42
C       USE EQUATION OF STATE FOR AIR **                                 E   43
      8 DO 9 J=J1,J2                                                     E   44
      9 GAMD(J)=GAMD(J)*D(J)                                             E   45
        IF (M-NMAT) 1,10,10                                              E   46
     10 RETURN                                                           E   47
        END                                                              E   48
```

```
      SUBROUTINE CSOUND                                                 F     1
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),   F     2
     .       PE(512),EE(512),TE(512),ANE(512),FM(512),CS(512)           F     3
      COMMON /RMAS/ ZM(512),ZMHI(512),RNH(512),RA(512),RANH(512)        F     4
      COMMON /MPROP/ PH(512),GAMD(512),CHI(512),BET(512),CI(512)        F     5
      COMMON /EQSF/ EXFAC(5 2),SQFAC(512)                               F     6
      COMMON /PMAT/ THIK(4),RAT(4),DV(4),RHO1(4),RHO(4),GRUN(4),HUGC(4),F     7
     .       HUGD(4),HUGS(4),GAM1(4),ESUB(4),ERAT(4),GAMC(4),AMWT(4),   F     8
     .       CHIO(4),BETA(4),AMU(4),AEX(4),ACL(4),AEXO(4),TOE(4),TVAP(4)F     9
     .       ,CVS(4),CVG(4),CVSI(4),CVGI(4),VIONE(4),VIONK(4),SIGIE(4), F    10
     .       SIGIK(4),VIBOE(4),VIBOK(4),VIBWE(4),VIBWK(4),VHNUE(4),     F    11
     .       HNUSQ(4),SIGV(4),SEO(4),SEC(4),WFUNE(4),WFUNK(4),DNM(4)    F    12
      COMMON /MPAR/ NMAT,MMOD(7),JBND(4),IZT(4),NZ(4)                   F    13
      COMMON /JPAR/ JOR,JORP,JACM,JAC,JACP,JTMAX,IALPM,JTSTAB,JTACC,    F    14
     .       JTEDP,JSR                                                  F    15
      COMMON /CTIME/ TSTOP,TIME,DTN,DTNH,DTNHS,DTSTAB,DTACC,DTEDP,ALPHA,F    16
     .       TMAX,TEMP,PRES,EINTI,EDPT,EDP,SOS                          F    17
      M=0                                                               F    18
      J2=1                                                              F    19
    1 M=M+1                                                             F    20
      J1=J2+1                                                           F    21
      J2=MIN0(JAC,JBND(M))                                              F    22
      IF (MMOD(M)) 2,3,5                                                F    23
C     CALL GILMORE EQUATION OF STATE FOR AIR **                         F    24
    2 CALL AIREST (M,J1,J2)                                             F    25
      IF (M-NMAT) 1,13,13                                               F    26
C     IDEAL GAS EQUATION OF STATE **                                    F    27
    3 DO 4 J=J1,J2                                                      F    28
      CS(J)=SQRT(GAMC(M)*E(J))                                          F    29
      T(J)=E(J)/CVG(M)                                                  F    30
    4 CHI(J)=CHIO(M)*T(J)**BETA(M)                                      F    31
      IF (M-NMAT) 1,13,13                                               F    32
C     HUGONOIT EQUATION OF STATE **                                     F    33
    5 DO 12 J=J1,J2                                                     F    34
      EMU=D(J)/RHO(M)-1.                                                F    35
      IF (EMU) 6,8,8                                                    F    36
```

```
C       FOR EXPANDED ZONES **                                                   F   37
    6   V1=RHO(M)/D(J)                                                          F   38
        ENU3=ERAT(M)*(1.-2.*V1)*V1                                              F   39
        CSQ=P(J)/D(J)+0.5*SQFAC(J)*(E(J)-ESUB(M)+EXFAC(J))-(GAM1(M)+            F   40
       .        SQFAC(J))*ENU3*EXFAC(J)                                         F   41
        IF (CSQ) 7,7,9                                                          F   42
    7   CSQ=GAM1(M)*(1.+GAM1(M))*E(J)                                           F   43
        GO TO 9                                                                 F   44
C       FOR COMPRESSED ZONES **                                                 F   45
    8   CSQ=GRUN(M)*E(J)+((3.*HUGS(M)*EMU+2.*HUGD(M))*EMU+HUGC(M)-GAM1(M)*      F   46
       .        ((2.*HUGS(M)*EMU+1.5*HUGD(M))*EMU+HUGC(M))*EMU)/RHO(M)          F   47
    9   CS(J)=SQRT(CSQ)                                                         F   48
        IF (ESUB(M)-E(J)) 10,11,11                                              F   49
   10   CI(J)=CVGI(M)                                                           F   50
        T(J)=TVAP(M)+(E(J)-ESUB(M))*CI(J)                                       F   51
        GO TO 12                                                                F   52
   11   CI(J)=CVSI(M)                                                           F   53
        T(J)=AMIN1(TVAP(M),TOE(M)+E(J)*CI(J))                                   F   54
   12   CHI(J)=CHIO(M)*T(J)**BETA(M)                                            F   55
        IF (M-NMAT) 1,13,13                                                     F   56
C       COMPUTE TIME STEP FOR THERMAL CONDUCTIVITY **                           F   57
   13   DTACC=0.                                                                F   58
        TJP=T(2)                                                                F   59
        YJP=R(2)/CHI(2)                                                         F   60
        XTRP=0.                                                                 F   61
        DO 16 J=2,JACM                                                          F   62
        TJM=TJP                                                                 F   63
        YJM=YJP                                                                 F   64
        XTRM=XTRP                                                               F   65
        YJP=(R(J+1)-R(J))/CHI(J+1)                                              F   66
        DEN=YJM+YJP                                                             F   67
        TJP=(T(J)*YJP+T(J+1)*YJM)/DEN                                           F   68
        XTRP=RA(J)*(T(J+1)-T(J))/DEN                                            F   69
        DT=ABS(TJP-TJM)                                                         F   70
        IF (DT-0.01*TJP) 16,16,14                                               F   71
   14   DTI=ABS(XTRP-XTRM)*CI(J)/(ZM(J)*DT)                                     F   72
```

```
      IF (DTI-DTACC) 16,16,15                                          F    73
15    DTACC=DTI                                                        F    74
      JTACC=J                                                          F    75
16    CONTINUE                                                         F    76
      IF (DTACC-1.) 17,17,18                                           F    77
17    DTACC=1.                                                         F    78
      RETURN                                                           F.   79
18    DTACC=0.125/DTACC                                                F    80
      RETURN                                                           F    81
      END                                                              F    82
```

```
      SUBROUTINE AIREST (M,J1,J2)                                          G     1
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),      G     2
     .       PE(512),EEL(512),TE(512),ANE(512),FM(512),CS(512)             G     3
      COMMON /MPROP/ PH(512),GAMD(512),CHI(512),BET(512),CI(512)           G     4
      COMMON /PMAT/ THIK(4),RAT(4),DV(4),RHO1(4),RHO(4),GRUN(4),HUGC(4),   G     5
     .       HUGD(4),HUGS(4),GAM1(4),ESUB(4),ERAT(4),GAMC(4),AMWT(4),      G     6
     .       CHI0(4),BETA(4),AMU(4),AEX(4),ACL(4),AEXO(4),TOE(4),TVAP(4)   G     7
     .       ,CVS(4),CVG(4),CVSI(4),CVGI(4),VIONE(4),VIONK(4),SIGIE(4),    G     8
     .       SIGIK(4),VIBOE(4),VIBOK(4),VIBWE(4),VIBWK(4),VHNUE(4),        G     9
     .       HNUSQ(4),SIGV(4),SEO(4),SEC(4),WFUNE(4),WFUNK(4),DNM(4)       G    10
      DIMENSION EE(10,8),TT(10),DTT(10),RD(8)                              G    11
      DIMENSION GAM(10,8),TC(4),DTCI(4),COND(5),BC(4)                      G    12
      DATA TCO,TC/2000.,3200.,4100.,7000.,9000./                           G    13
      DATA DTCI/.83333333E-03,1.1111111E-03,.3448276E-03,.5E-03/           G    14
      DATA TT/2000.,3000.,4000.,5000.,6000.,7000.,8000.,12000.,18000.,     G    15
     .       24000./                                                       G    16
      DATA DTT/1000.,1000.,1000.,1000.,1000.,1000.,1000.,4000.,6000.,      G    17
     .       6000./                                                        G    18
      DATA COND/11787.,70000.,45000.,350000.,141568./                      G    19
      DATA CONE,BE/1.3682E-05,2.5/                                         G    20
      DATA BC/3.88,-1.84,3.81,-3.60/                                       G    21
      DATA RD/-6.,-5.,-4.,-3.,-2.,-1.,0.,1./                               G    22
      DATA EE/.25283E11,.63209E11,.12956E12,.32257E12,.35229E12,           G    23
     .       .41705E12,.62957E12,.14559E13,.21642E13,.38892E13,            G    24
     .       .19867E11,.61743E11,.91757E11,.26456E12,.34438E12,            G    25
     .       .37620E12,.46088E12,.14023E13,.16849E13,.35347E13,            G    26
     .       .17929E11,.59274E11,.78111E11,.17020E12,.32048E12,            G    27
     .       .36100E12,.39943E12,.11419E13,.15911E13,.25865E13,            G    28
     .       .17297E11,.50106E11,.73171E11,.11382E12,.23927E12,            G    29
     .       .34053E12,.37661E12,.76269E12,.15455E13,.18866E13,            G    30
     .       .17096E11,.38122E11,.69488E11,.91632E11,.15396E12,            G    31
     .       .26769E12,.34899E12,.54795E12,.13349E13,.16991E13,            G    32
     .       .17029E11,.31667E11,.60990E11,.82506E11,.11231E12,            G    33
     .       .17552E12,.26803E12,.46423E12,.93725E12,.15049E13,            G    34
     .       .17008E11,.29239E11,.49897E11,.73757E11,.94813E11,            G    35
     .       .12591E12,.17786E12,.41944E12,.67395E12,.11089E13,            G    36
```

```
     .        .16999E11,.28444E11,.43618E11,.62874E11,.82673E11,          G   37
     .        .10344E12,.13081E12,.33367E12,.56176E12,.82046E12/          G   38
      DATA GAM/.23810,.16500,.12310,.08699,.09805,.10170,.09126,.09393,   G   39
     .        .10820,.10510,.29410,.16840,.15910,.09635,.09939,.10840,    G   40
     .        .16810,.09495,.12510,.10960,.32210,.17310,.18070,.12510,    G   41
     .        .10290,.11120,.11780,.10160,.12940,.12730,.33260,.19500,    G   42
     .        .19000,.16500,.11980,.11440,.12210,.11910,.13080,.15130,    G   43
     .        .33610,.23980,.19670,.19380,.15670,.12890,.12680,.13980,    G   44
     .        .13620,.15940,.33750,.27800,.21350,.20870,.19440,.16390,    G   45
     .        .14390,.15300,.15310,.16500,.33700,.29670,.24470,.22290,    G   46
     .        .21780,.20280,.18080,.16080,.17510,.17810,.33770,.30360,    G   47
     .        .26940,.24490,.23500,.22860,.21760,.17640,.18980,.19500/    G   48
      DATA EEO,ALDO/7.551544E+09,-2.888637/                               G   49
      DO 7 J=J1,J2                                                        G   50
      IF (E(J)-EEO) 1,1,2                                                 G   51
    1 CI(J)=CVGI(M)                                                       G   52
      T(J)=CI(J)*E(J)                                                     G   53
      GAMD(J)=GAM1(M)                                                     G   54
      CS(J)=SQRT(GAMC(M)*E(J))                                            G   55
      GO TO 7                                                             G   56
    2 RDL=ALOG10(D(J))-ALDO                                               G   57
      DO 3 K=2,8                                                          G   58
      IF (RD(K)-RDL) 3,4,4                                                G   59
    3 CONTINUE                                                            G   60
    4 BM=RD(K)-RDL                                                        G   61
      BP=1.-BM                                                            G   62
      EP=EEO                                                              G   63
      DO 5 I=1,10                                                         G   64
      EM=EP                                                               G   65
      EP=BM*EE(I,K-1)+BP*EE(I,K)                                          G   66
      IF (EP-E(J)) 5,6,6                                                  G   67
    5 CONTINUE                                                            G   68
    6 CI(J)=DTT(I)/(EP-EM)                                                G   69
      AM=(EP-E(J))/(EP-EM)                                                G   70
      AP=1.-AM                                                            G   71
      T(J)=TT(I)-DTT(I)*AM                                                G   72
```

```
      GAMD(J)=AP*BP*GAM(I,K)+AP*BM*GAM(I,K-1)+AM*BP*GAM(I-1,K)+AM*BM*      G    73
     .       GAM(I-1,K-1)                                                  G    74
      CS(J)=SQRT(GAMD(J)*(1.+GAMD(J))*E(J))                               G    75
    7 CONTINUE                                                            G    76
      IF (CHIO(M)) 14,14,8                                                G    77
    8 DO 13 J=J1,J2                                                       G    78
      IF (T(J)-TCO) 9,9,10                                                G    79
    9 CHI(J)=CHIO(M)*T(J)**BETA(M)                                        G    80
      BET(J)=BETA(M)                                                      G    81
      GO TO 13                                                            G    82
   10 DO 11 K=1,4                                                         G    83
      IF (T(J)-TC(K)) 12,12,11                                            G    84
   11 CONTINUE                                                            G    85
      CHI(J)=CONE*SQRT(T(J))*T(J)**2                                      G    86
      BET(J)=BE                                                           G    87
      GO TO 13                                                            G    88
   12 AM=(TC(K)-T(J))*DTCI(K)                                             G    89
      CHI(J)=AM*COND(K)+(1.-AM)*COND(K+1)                                 G    90
      BET(J)=BC(K)                                                        G    91
   13 CONTINUE                                                            G    92
   14 RETURN                                                              G    93
      END                                                                 G    94
```

```
      SUBROUTINE START                                                    H     1
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),     H     2
     .       PE(512),EE(512),TE(512),ANE(512),EM(512),CS(512)             H     3
      COMMON /RMAS/ ZM(512),ZMHI(512),RNH(512),RA(512),RANH(512)          H     4
      COMMON /MPROP/ PH(512),GAMD(512),CHI(512),BET(512),CI(512)          H     5
      COMMON /PMAT/ THIK(4),RAT(4),DV(4),RHO1(4),RHO(4),GRUN(4),HUGC(4),  H     6
     .       HUGD(4),HUGS(4),GAM1(4),ESUB(4),ERAT(4),GAMC(4),AMWI(4),     H     7
     .       CHIO(4),BETA(4),AMU(4),AEX(4),ACL(4),AEXO(4),TOE(4),TVAP(4)  H     8
     .       ,CVS(4),CVG(4),CVSI(4),CVGI(4),VIONE(4),VIONK(4),SIGIE(4),   H     9
     .       SIGIK(4),VIBOE(4),VIBOK(4),VIBWE(4),VIBWK(4),VHNUE(4),       H    10
     .       HNUSQ(4),SIGV(4),SEO(4),SEC(4),WFUNE(4),WFUNK(4),DNM(4)      H    11
      COMMON /MPAR/ NMAT,MMOD(7),JBND(4),IZT(4),NZ(4)                     H    12
      COMMON /JPAR/ JOR,JORP,JACM,JAC,JACP,JTMAX,IALPM,JTSTAB,JTACC,      H    13
     .       JTEDP,JSR                                                    H    14
      COMMON /NTIME/ NCYCS,N,NELOP,NEL,NEDIT,LINE,LINMX,NTAPE,NPRIN,      H    15
     .       IPRIN,JINT,JMIN,JMAX,NTED,KTAPE,NETP                        H    16
      COMMON /CTIME/ TSTOP,TIME,DTN,DTNH,DTNHS,DTSTAB,DTACC,DTEDP,ALPHA,  H    17
     .       TMAX,TEMP,PRES,EINTI,EDPT,EDP,SDS                            H    18
      COMMON /NAME/ IDT(12),MATN(8)                                       H    19
      COMMON /LASER/ POW,WL,WF,CRITE,EC                                   H    20
      COMMON /EPAR/ ACPAO,REC,SIGO,SIGC,DIFI,ATIE,ALO,AKBT,AKB,GAME,      H    21
     .       FNEL,NEPRN                                                   H    22
      COMMON /EDTIME/ TEDIT,REDIT,DTEDT                                   H    23
      DATA AO,C,RMC2,RE,ALPHAI,PI/6.02252E+23,2.997925E+10,0.511006E+06,  H    24
     .       2.81777E-13,137.0388,3.1415927/                              H    25
      DATA AKB,GAME,RATM,SIGO/1.38046E-16,0.66666667,1836.,11.346/        H    26
      DATA AODEG,CTEV,TO,ATM/8.3143E+07,11605.7,277.144,1.01325E+06/      H    27
      DATA ABOHR,RYD/0.52967E-08,13.605/                                  H    28
C     READ INPUT PARAMETERS **                                            H    29
      READ 39, (IDT(I),I=1,12)                                            H    30
      READ 40, NCYCS,NMAT,IALPH,NEDIT,LINMX,INPUT,NELOP,NEPRN             H    31
      READ 40, NTAPE,NPRIN,IPRIN,JINT,JMIN,JMAX                           H    32
      READ 41, TSTOP,DTMIN,TEDIT,REDIT,DTEDT,TMAX,RTMAX                   H    33
      LINMX=MAX0(LINMX,30)                                                H    34
      JMIN=MAX0(JMIN,1)                                                   H    35
      FNEL=4.1/FLOAT(NELOP)                                               H    36
```

```
      IF (INPUT) 1,1,30                                              H   37
C     INPUT FROM CARDS **                                            H   38
    1 READ 41, POWW,WLM,TEMP,PRESA                                   H   39
      DO 3 M=1,NMAT                                                  H   40
      READ 43, IZT(M),NZ(M),THIK(M),RAT(M)                           H   41
      IF (RAT(M)) 2,2,3                                              H   42
    2 RAT(M)=1.0                                                     H   43
    3 CONTINUE                                                       H   44
C     CHECK INPUT PARAMETERS AND CHANGE UNITS **                     H   45
      IF (TEMP) 4,4,5                                                H   46
    4 TEMP=TO                                                        H   47
    5 IF (PRESA) 6,6,7                                               H   48
    6 PRESA=1.0                                                      H   49
    7 IF (TMAX) 8,8,9                                                H   50
    8 TMAX=TO+TO                                                     H   51
    9 PRES=PRESA*ATM                                                 H   52
      POW=POWW*1.E+07                                                H   53
      WL=WLM*1.E-04                                                  H   54
      IALPM=IALPH-1                                                  H   55
      ALPHA=FLOAT(IALPH)                                             H   56
      ALPHI=1./ALPHA                                                 H   57
      MMOD(7)=1                                                      H   58
C     COMPUTE INITIAL ZONING **                                      H   59
      R2=0.                                                          H   60
      JOR=1                                                          H   61
      DO 17 M=1,NMAT                                                 H   62
      R1=R2                                                          H   63
      R2=R2+THIK(M)                                                  H   64
      IF (IZT(M)-2) 10,11,11                                         H   65
   10 V1=R1                                                          H   66
      V2=R2                                                          H   67
      ALPIZ=1.                                                       H   68
      GO TO 12                                                       H   69
   11 V1=R1**IALPH                                                   H   70
      V2=R2**IALPH                                                   H   71
      ALPIZ=ALPHI                                                    H   72
```

```
   12 J1=JOR+1                                                          M   73
      JOR=JOR+NZ(M)-1                                                   M   74
      IF (RAT(M)-1.0) 13,14,13                                          M   75
   13 DV(M)=(V2-V1)*(RAT(M)-1.)/(RAT(M)**NZ(M)-1.)                      M   76
      GO TO 15                                                          M   77
   14 DV(M)=(V2-V1)/FLOAT(NZ(M))                                        M   78
   15 DVV=DV(M)                                                         M   79
      VR=V1                                                             M   80
      DO 16 J=J1,JOR                                                    M   81
      VR=VR+DVV                                                         M   82
      R(J)=VR**ALPIZ                                                    M   83
      RNH(J)=R(J)+R(J)                                                  M   84
      RA(J)=R(J)**IALPM                                                 M   85
      RANH(J)=RA(J)+RA(J)                                               M   86
   16 DVV=DVV*RAT(M)                                                    M   87
      JOR=JOR+1                                                         M   88
      JBND(M)=JOR                                                       M   89
      R(JOR)=R2                                                         M   90
      RNH(JOR)=R(JOR)+R(JOR)                                            M   91
      RA(JOR)=R(JOR)**IALPM                                             M   92
   17 RANH(JOR)=RA(JOR)+RA(JOR)                                         M   93
      JORP=JOR+1                                                        M   94
      VR=V2+DVV*RAT(M)                                                  M   95
      R(JORP)=VR**ALPIZ                                                 M   96
      RNH(JORP)=R(JORP)+R(JORP)                                         M   97
      RA(JORP)=R(JORP)**IALPM                                           M   98
      RANH(JORP)=RA(JORP)+RA(JORP)                                      M   99
      DO 18 J=1,JORP                                                    M  100
      IF (R(J)-RTMAX) 18,19,19                                          M  101
   18 CONTINUE                                                          M  102
   19 JTMAX=J                                                           M  103
C     COMPUTE ELECTRON SCATTERING AND ABSORPTION COEFFICIENTS **        M  104
      RO=RYD*CTEV                                                       M  105
      ACPA0=2.*C*PI*ABOHR**2/(ALPHAI*SQRT(PI*RO))                       M  106
      REC=4.*ABOHR**3*PI*RO*SQRT(PI*RO)                                 M  107
      SIGC=2.*RO**2                                                     M  108
```

```
      DIFI=RO*(ALPHAI/C)**2                                                    M  109
      AKBT=AKB/GAME                                                            M  110
      ATIE=0.125*C/(ALPHAI*ABOHR*(RO*PI*ABOHR)**2)                             M  111
      ATIE=ALOG(ATIE)                                                          M  112
      ALO=-ALOG(32.*PI*(ABOHR*RO)**3/9.)                                       M  113
      WF=2.*PI*C/WL                                                            M  114
      CRITE=(WF*ALPHAI/C)**2/(4.*PI*ABOHR)                                     M  115
      EC=RE*WL**2/(PI*C)                                                       M  116
      J2=0                                                                     M  117
      M=0                                                                      M  118
C     READ MATERIAL PROPERTIES **                                              M  119
   20 M=M+1                                                                    M  120
      M2=2*M                                                                   M  121
      M1=M2-1                                                                  M  122
      READ 42, MATN(M1),MATN(M2),MMOD(M),RHO(M),GAM1(M),AMWT(M),VIONE(M)       M  123
     .     ,SIGIE(M)                                                         M  124
      READ 41, CHIO(M),RETA(M),ACL(M),AEXO(M),VIBOE(M),VIBWE(M),VHNUE(M)       M  125
     .     ,SIGV(M)                                                          M  126
      VIONK(M)=VIONE(M)*CTEV                                                   M  127
      SIGIK(M)=SIGIE(M)/CTEV                                                   M  128
      VIBOK(M)=VIBOE(M)*CTEV                                                   M  129
      VIBWK(M)=VIBWE(M)*CTEV                                                   M  130
      HNUSQ(M)=(VHNUE(M)*CTEV)**2                                              M  131
      SEO(M)=0.5*SIGO/(RATM*AMWT(M))                                           M  132
      SEC(M)=2.*SIGC/(RATM*AMWT(M))                                            M  133
      DNM(M)=AO/AMWT(M)                                                        M  134
      CVG(M)=AODEG/(AMWT(M)*GAM1(M))                                           M  135
      CVGI(M)=1./CVG(M)                                                        M  136
      IF (MMOD(M)) 21,21,22                                                    M  137
   21 GAMC(M)=GAM1(M)*(GAM1(M)+1.)                                             M  138
      CVS(M)=CVG(M)                                                            M  139
      GRUN(M)=GAM1(M)                                                          M  140
      EJ=CVG(M)*TEMP                                                           M  141
      RHO1(M)=PRES/(GRUN(M)*CVG(M)*TEMP)                                       M  142
      GO TO 23                                                                 M  143
   22 READ 41, GRUN(M),HUGC(M),HUGD(M),HUGS(M),ESUB(M),CVS(M),TVAP(M),         M  144
```

```
      WFUNE(M)                                                          H  145
      WFUNK(M)=CTEV*WFUNE(M)                                            H  146
      ERAT(M)=HUGC(M)/(GRUN(M)*ESUB(M)*RHO(M))                          H  147
      EJ=ATM/(GRUN(M)*RHO(M))                                           H  148
      TOE(M)=TO-EJ/CVS(M)                                               H  149
      EJ=CVS(M)*(TEMP-TOE(M))                                           H  150
      RHO1(M)=RHO(M)*(HUGC(M)+PRES)/(HUGC(M)+GRUN(M)*RHO(M)*EJ)         H  151
   23 AEX(M)=AEXO(M)+1.                                                 H  152
      AMU(M)=ACL(M)/RHO(M)**AEX(M)                                      H  153
      CVSI(M)=1./CVS(M)                                                 H  154
      CHIJ=CHIO(M)*TEMP**BETA(M)                                        H  155
C     SET INITIAL VALUES INTO MESH **                                   H  156
      J1=J2+1                                                           H  157
      J2=JBND(M)+M/NMAT                                                 H  158
      DO 24 J=J1,J2                                                     H  159
      D(J)=RHO1(M)                                                      H  160
      E(J)=EJ                                                           H  161
      T(J)=TEMP                                                         H  162
      TE(J)=TEMP                                                        H  163
      CHI(J)=CHIJ                                                       H  164
      CI(J)=CVSI(M)                                                     H  165
      GAMD(J)=GRUN(M)                                                   H  166
   24 BET(J)=BETA(M)                                                    H  167
      IF (M-NMAT) 20,25,25                                              H  168
   25 JAC=JOR                                                           H  169
      CALL EOST                                                         H  170
      J2=1                                                              H  171
      DO 26 M=1,NMAT                                                    H  172
      J1=J2+1                                                           H  173
      J2=JBND(M)+M/NMAT                                                 H  174
      DO 26 J=J1,J2                                                     H  175
      P(J)=PH(J)+GAMD(J)*E(J)                                           H  176
      GAMD(J)=GRUN(M)                                                   H  177
      ZM(J)=ALPHT*RHO1(M)*(RA(J)*R(J)-RA(J-1)*R(J-1))                   H  178
   26 ZMHI(J-1)=1./(ZM(J-1)+ZM(J))                                      H  179
      P(1)=P(2)                                                         H  180
```

```
      P(JORP)=P(JOR)                                                    H  181
      DTNH=DTMIN                                                        H  182
      DTN=DTMIN                                                         H  183
      DTNHS=DTMIN                                                       H  184
      JACM=JBND(1)                                                      H  185
      JAC=JACM+1                                                        H  186
      JACP=JAC+1                                                        H  187
C     DETERMINE SURFACE OF SOLID PARTICLE **                            H  188
      J2=1                                                              H  189
      DO 27 M=1,NMAT                                                    H  190
      J1=J2+1                                                           H  191
      J2=JBND(M)                                                        H  192
      DO 27 J=J1,J2                                                     H  193
      IF (E(J)-ESUB(M)) 27,27,28                                        H  194
   27 CONTINUE                                                          H  195
   28 JSR=J-1                                                           H  196
C     COMPUTE INITIAL INTERNAL ENERGY **                                H  197
      SOS=0.                                                            H  198
      EDPT=0.                                                           H  199
      EINTI=0.                                                          H  200
      DO 29 J=2,JOR                                                     H  201
   29 EINTI=EINTI+E(J)*ZM(J)                                            H  202
      GO TO 35                                                          H  203
C     READ INPUT FROM TAPE **                                           H  204
   30 DO 34 IN=1,INPUT                                                  H  205
      READ (8) (IDT(I),I=1,20),KTAPE,JOR,JORP,JACM,JAC,JACP,JTMAX,JSR,  H  206
     .       IALPM,NMAT,(MMOD(J),J=1,19),ALPHA,TMAX,TEMP,PRES,EINTI,    H  207
     .       EDPT,POW,WL,WF,CRITE,ACPAO,REC,SIGC,DIFI,ATIE,ALO,AKBT,    H  208
     .       (THIK(K),K=1,168)                                          K  209
      IF (KTAPE-1) 33,33,31                                             H  210
   31 KDO=KTAPE-1                                                       H  211
      DO 32 K=1,KDO                                                     H  212
   32 READ (8) TIME,DTNHS                                               H  213
   33 READ (8) TIME,DTNHS,DTNH,N,JORP,(R(J),P(J),Q(J),U(J),D(J),E(J),   H  214
     .       T(J),PE(J),EE(J),TE(J),ANE(J),FM(J),CS(J),J=1,JORP)        H  215
   34 READ (8) DTN,DTNH,(PH(J),GAMD(J),CHI(J),BET(J),CI(J),ZM(J),ZMHI(J) H  216
```

```
     .       ,RNH(J),RA(J),RANH(J),J=1,JORP)                                  H  217
      REWIND 8                                                                H  218
      POWW=POW*1.E-07                                                         H  219
      WLM=WL*1.E+04                                                           H  220
      PRESA=PRES/ATM                                                          H  221
      NCYCS=N+NCYCS                                                           H  222
C     PRINT INPUT PARAMETERS **                                               H  223
   35 PRINT 48, (IDT(I),I=1,12)                                               H  224
      PRINT 49                                                                H  225
      PRINT 44, NCYCS,NMAT,IALPH,NEOIT,LINMX,INPUT,NELOP,NEPRN                H  226
      PRINT 50                                                                H  227
      PRINT 44, NTAPE,NPRIN,IPRIN,JINT,JMIN,JMAX                              H  228
      PRINT 51                                                                H  229
      PRINT 45, TSTOP,DTMIN,TEOIT,REDIT,DTEDT,TMAX,RTMAX                      H  230
      PRINT 52                                                                H  231
      PRINT 45, POWW,WLM,TEMP,PRESA                                           H  232
      PRINT 53                                                                H  233
      PRINT 54                                                                H  234
      DO 36 M=1,NMAT                                                          H  235
   36 PRINT 46, M,IZT(M),NZ(M),THIK(M),RAT(M),DV(M)                           H  236
      DO 38 M=1,NMAT                                                          H  237
      M2=2*M                                                                  H  238
      M1=M2-1                                                                 H  239
      PRINT 47                                                                H  240
      PRINT 55, MATN(M1),MATN(M2),MMOD(M)                                     H  241
      PRINT 56                                                                H  242
      PRINT 45, RHO(M),GAM1(M),AMWT(M),CHIO(M),BETA(M),ACL(M),AEXO(M)         H  243
      PRINT 58                                                                H  244
      PRINT 45, VIONE(M),SIGIE(M),VIBOE(M),VIBWE(M),VHNUE(M),SIGV(M)          H  245
      IF (MMOD(M)) 38,38,37                                                   H  246
   37 PRINT 57                                                                H  247
      PRINT 45, GRUN(M),HUGC(M),HUGD(M),HUGS(M),ESUB(M),ERAT(M),CVS(M),       H  248
     .       TVAP(M),WFUNE(M)                                                 H  249
   38 CONTINUE                                                                H  250
      WRITE (9) (IOT(I),I=1,12),POW,WL,TEMP,PRES,JOR                          H  251
      RETURN                                                                  H  252
```

```
39 FORMAT (12A5)                                                          H  253
40 FORMAT (8I10)                                                          H  254
41 FORMAT (8E10.4)                                                        H  255
42 FORMAT (2A5,I10,6E10.4)                                                H  256
43 FORMAT (2I10,7E10.4)                                                   H  257
44 FORMAT (I8,10I12)                                                      H  258
45 FORMAT (10E12.3)                                                       H  259
46 FORMAT (I8,2I12,4X,8E12.3)                                             H  260
47 FORMAT (///)                                                           H  261
48 FORMAT (1H1,3X,12A5,/)                                                 H  262
49 FORMAT (/,4X,6H NCYCS,7X,5H NMAT,6X,6H IALPH,6X,6H NEDIT,6X,           H  263
  .       6H LINMX,6X,6H INPUT,6X,6H NELOP,6X,6H NERPN)                   H  264
50 FORMAT (/,4X,6H NTAPE,6X,6H NPRIN,6X,6H IPRIN,6X,5H JINT,7X,           H  265
  .       5H JMIN,7X,5H JMAX)                                             H  266
51 FORMAT (/,4X,6H TSTOP,6X,6H DTMIN,6X,6H TEDIT,6X,6H REDIT,6X,          H  267
  .       6H DTEDT,7X,5H TMAX,6X,6H RTMAX)                                H  268
52 FORMAT (/,5X,4H POW,8X,3H WL,8X,5H TEMP,7X,5H PRES)                    H  269
53 FORMAT (//,4X,48H ZONING DETERMINED BY RADIUS AND NUMBER OF ZONES)     H  270
54 FORMAT (/,6X,2H M,9X,4H IZT,8X,3H NZ,8X,5H THIK,8X,4H RAT,11X,         H  271
  .       2HDV)                                                           H  272
55 FORMAT (4X,25H MATERIAL PROPERTIES FOR ,2A5,8X,7H MMOD =,I2)           H  273
56 FORMAT (/,5X,4H RHO,7X,5H GAM1,7X,5H AMWT,7X,5H CHI0,7X,5H BETA,       H  274
  .       8X,4H ACL,8X,4H AEX)                                            H  275
57 FORMAT (/,4X,5H GRUN,7X,5H HUGC,7X,5H HUGD,7X,5H HUGS,7X,5H ESUB,      H  276
  .       7X,5H ERAT,8X,4H CVS,7X,5H TVAP,7X,5H WFUN)                     H  277
58 FORMAT (/,4X,5H VION,7X,5H SIGI,7X,5H VI80,7X,5H VIBW,7X,5H VHNU,      H  278
  .       7X,5H SIGV)                                                     H  279
   END                                                                    H  280
```

```
      SUBROUTINE EDITS                                                  I     1
C     COMPUTES MAXIMUM ELECTRON DENSITY **                              I     2
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),   I     3
     .     PE(512),EE(512),TE(512),ANE(512),FM(512),CS(512)             I     4
      COMMON /RMAS/ ZM(512),ZMHI(512),RNH(512),RA(512),RANH(512)        I     5
      COMMON /MPROP/ PH(512),GAMD(512),CHI(512),BET(512),CI(512)        I     6
      COMMON /PMAT/ THIK(4),RAT(4),DV(4),RHO1(4),RHO(4),GRUN(4),HUGC(4),I     7
     .     HUGD(4),HUGS(4),GAM1(4),ESUB(4),ERAT(4),GAMC(4),AMWT(4),     I     8
     .     CHIO(4),BETA(4),AMU(4),AEX(4),ACL(4),AEXO(4),TOE(4),TVAP(4)  I     9
     .     ,CVS(4),CVG(4),CVSI(4),CVGI(4),VIONE(4),VIONK(4),SIGIE(4),   I    10
     .     SIGIK(4),VIBOE(4),VIBOK(4),VIBWE(4),VIBWK(4),VHNUE(4),       I    11
     .     HNUSQ(4),SIGV(4),SEO(4),SEC(4),WFUNE(4),WFUNK(4),DNM(4)      I    12
      COMMON /MPAR/ NMAT,MMOD(7),JBND(4),IZT(4),NZ(4)                   I    13
      COMMON /JPAR/ JOR,JORP,JACM,JAC,JACP,JTMAX,IALPM,JTSTAB,JTACC,    I    14
     .     JTEDP,JSR                                                    I    15
      COMMON /NTIME/ NCYCS,N,NELOP,NEL,NEDIT,LINE,LINMX,NTAPE,NPRIN,    I    16
     .     IPRIN,JINT,JMIN,JMAX,NTED,KTAPE,NETP                        I    17
      COMMON /CTIME/ TSTOP,TIME,DTN,DTNH,DTNHS,DTSTAB,DTACC,DTEDP,ALPHA,I    18
     .     TMAX,TEMP,PRES,EINTI,EDPT,EDP,SOS                            I    19
      PMAX=0.                                                           I    20
      ANMAX=0.                                                          I    21
      ANAVG=0.                                                          I    22
      VOL=0.                                                            I    23
      EKIN=0.                                                           I    24
      EINT=0.                                                           I    25
      DO 7 J=2,JAC                                                      I    26
      IF (PMAX-P(J)) 1,2,2                                              I    27
    1 PMAX=P(J)                                                         I    28
      JPMAX=J                                                           I    29
    2 IF (ANE(J)) 4,4,3                                                 I    30
    3 DV=ZM(J)/D(J)                                                     I    31
      ANAVG=ANAVG+ANE(J)*DV                                             I    32
      VOL=VOL+DV                                                        I    33
    4 IF (ANMAX-ANE(J)) 5,6,6                                           I    34
    5 ANMAX=ANE(J)                                                      I    35
      JNMAX=J                                                           I    36
```

```
    6 EINT=EINT+(E(J)+EE(J))*ZM(J)                                          I    37
      PMV=U(J)*(ZM(J)+ZM(J+1))                                              I    38
    7 EKIN=EKIN+U(J)*PMV                                                    I    39
      IF (JAC-JOR) 8,10,10                                                  I    40
    8 DO 9 J=JACP,JOR                                                       I    41
    9 EINT=EINT+E(J)*ZM(J)                                                  I    42
   10 PMVNEG=0.5*PMVNEG                                                     I    43
      PMVPOS=0.5*PMVPOS                                                     I    44
      EKIN=0.25*EKIN                                                        I    45
      ENEW=EKIN+EINT-EINTI                                                  I    46
      IF (VOL) 12,12,11                                                     I    47
   11 ANAVG=ANAVG/VOL                                                       I    48
   12 PRINT 19                                                              I    49
      PRINT 16, N,JAC,JTSTAB,JTACC,JTEDP,JPMAX,NEL,JNMAX,EDP,EKIN,ENEW      I    50
      PRINT 20                                                              I    51
      PRINT 17, TIME,DTNH,DTSTAB,DTACC,DTEDP,PMAX,SOS,ANMAX,ANAVG,EINT,     I    52
     .       EDPT                                                           I    53
      LINE=LINE+6                                                           I    54
      IF (LINMX-LINE) 13,14,15                                              I    55
   13 PRINT 18                                                              I    56
   14 LINE=6                                                                I    57
   15 RETURN                                                                I    58
   16 FORMAT (I8,7I12,4X,3E12.3)                                            I    59
   17 FORMAT (11E12.3,/)                                                    I    60
   18 FORMAT (1H1)                                                          I    61
   19 FORMAT (/,6X,2H N,8X,4H JAC,7X,7H JTSTAB,6X,6H JTACC,6X,6H JTEDP,     I    62
     .       6X,6H JPMAX,6X,6H  NEL ,6X,6H JNMAX,5X,7H   EDP ,7X,          I    63
     .       5H EKIN,7X,5H ENEW)                                            I    64
   20 FORMAT (4X,5H TIME,7X,5H DTNH,6X,7H DTSTAB,6X,6H DTACC,6X,            I    65
     .       6H DTEDP,7X,5H PMAX,7X,5H SOS ,6X,6H ANMAX,5X,7H  ANAVG,7X,    I    66
     .       5H EINT,7X,5H EDPT)                                            I    67
      END                                                                   I    68
```

```
      SUBROUTINE FINED                                                  J     1
      COMMON /RVAR/ R(512),P(512),Q(512),U(512),D(512),E(512),T(512),   J     2
     .       PE(512),EE(512),TE(512),ANE(512),FM(512),CS(512)           J     3
      COMMON /RMAS/ ZM(512),ZMHI(512),RNH(512),RA(512),RANH(512)        J     4
      COMMON /MPROP/ PH(512),GAMD(512),CHI(512),BET(512),CI(512)        J     5
      COMMON /PMAT/ THIK(4),RAT(4),DV( ),RHO1(4),RHO(4),GRUN(4),HUGC(4),J     6
     .       HUGD(4),HUGS(4),GAM1(4),ESUB(4),ERAT(4),GAMC(4),AMWT(4),   J     7
     .       CHIO(4),BETA(4),AMU(4),AEX(4),ACL(4),AEXO(4),TOE(4),TVAP(4)J     8
     .       ,CVS(4),CVG(4),CVSI(4),CVGI(4),VIONE(4),VIONK(4),SIGIE(4), J     9
     .       SIGIK(4),VIBOE(4),VIBOK(4),VIBWE(4),VIBWK(4),VHNUE(4),     J    10
     .       HNUSQ(4),SIGV(4),SEO(4),SEC(4),WFUNE(4),WFUNK(4),DNM(4)    J    11
      COMMON /MPAR/ NMAT,MMOD(7),JBND(4),IZT(4),NZ(4)                   J    12
      COMMON /JPAR/ JOR,JORP,JACM,JAC,JACP,JTMAX,IALPM,JTSTAB,JTACC,    J    13
     .       JTEDP,JSR                                                  J    14
      COMMON /NTIME/ NCYCS,N,NELOP,NEL,NEDIT,LINE,LINMX,NTAPE,NPRIN,    J    15
     .       IPRIN,JINT,JMIN,JMAX,NTED,KTAPE,NETP                       J    16
      COMMON /CTIME/ TSTOP,TIME,DTN,DTNH,DTNHS,DTSTAB,DTACC,DTEDP,ALPHA,J    17
     .       TMAX,TEMP,PRES,EINTI,EDPT,EDP,SOS                          J    18
      COMMON /NAME/ IDT(12),MATN(8)                                     J    19
      COMMON /LASER/ POW,WL,WF,CRITE,EC                                 J    20
      COMMON /EPAR/ ACPAO,REC,SIGO,SIGC,DIFI,ATIE,ALO,AKBT,AKB,GAME,    J    21
     .       FNEL,NEPRN                                                 J    22
      DIMENSION A(121)                                                  J    23
      DATA AMICRN,BARI,BLANK,DASH,PLUS/1.E+04,1.E-06,1H ,1H-,1H+/       J    24
      DATA SMP,SMU,SMT,SMN,KP,KU,KT,KN/1HP,1HU,1HT,1HN,61,61,1,1/       J    25
      DATA SME/1HE/                                                     J    26
      IF (NTAPE) 1,2,1                                                  J    27
    1 WRITE (8,64) (IDT(I),I=1,20),KTAPE,JOR,JORP,JACM,JAC,JACP,JTMAX,  J    28
     .       JSR,IALPM,NMAT,(MMOD(J),J=1,19),ALPHA,TMAX,TEMP,PRES,EINTI,J    29
     .       EDPT,POW,WL,WF,CRITE,ACPAO,REC,SIGC,DIFI,ATIE,ALO,AKBT,    J    30
     .       (THIK(K),K=1,168)                                          J    31
    2 NEPR=0                                                            J    32
      IF (NPRIN) 3,5,4                                                  J    33
    3 NEPR=NTED                                                         J    34
      GO TO 5                                                           J    35
    4 NEPR=1                                                            J    36
```

```
 5 READ (9) (IDT(I),I=1,12),POW,WL,TEMP,PRES,JOR                    J   37
   IF (JINT) 7,7,6                                                  J   38
 6 JLIN=2+(JMAX-JMIN-1)/JINT                                        J   39
   JLIN=(JLIN-1)/6                                                  J   40
   JLIN=7+6*JLIN                                                    J   41
   JIN=JINT                                                         J   42
 7 DO 56 NE=1,NTED                                                  J   43
   READ (9) TIME,DTNHS,DTNH,N,JAC,(R(J),P(J),Q(J),U(J),D(J),E(J),T(J)  J   44
  .       ,PE(J),EE(J),TE(J),ANE(J),FM(J),CS(J),J=1,JAC)            J   45
   FM(J)=FM(J)/DTNH                                                 J   46
   P(J)=P(J)+Q(J)                                                   J   47
   IF (JINT) 8,8,11                                                 J   48
 8 IF (N) 1 ,10,9                                                   J   49
 9 JMAX=MAX0(JMAX,JAC)                                              J   50
10 JIN=1+(JMAX-JMIN)/54                                             J   51
   JLIN=2+(JMAX-JMIN-1)/JIN                                         J   52
   JLIN=(JLIN-1)/6                                                  J   53
   JLIN=7+6*JLIN                                                    J   54
11 IF (IPRIN-1) 12,12,55                                            J   55
12 PRINT 59, TIME,DTNHS,N,NE,(IDT(I),I=1,12)                        J   56
   JSIX=1                                                           J   57
   J=JMIN                                                           J   58
   DO 54 JL=1,JLIN                                                  J   59
   IF (JSIX-JL) 15,13,15                                            J   60
13 DO 14 K=1,111,10                                                 J   61
   A(K)=PLUS                                                        J   62
   DO 14 L=1,9                                                      J   63
14 A(K+L)=DASH                                                      J   64
   A(121)=PLUS                                                      J   65
15 IF (J-JOR) 17,17,16                                              J   66
16 RJ=0.                                                            J   67
   GO TO 48                                                         J   68
17 PA=BARI*ABS(P(J))                                                J   69
   IF (PA-1.12) 18,18,19                                            J   70
18 LP=KP                                                            J   71
   GO TO 22                                                         J   72
```

```
19 PB=10.*ALOG10(PA)+0.5                                          J   73
   L=INT(PB)                                                      J   74
   IF (P(J)) 20,20,21                                             J   75
20 LP=KP-L                                                        J   76
   GO TO 22                                                       J   77
21 LP=KP+L                                                        J   78
22 IF (T(J)-100.) 23,24,24                                        J   79
23 LT=KT                                                          J   80
   GO TO 25                                                       J   81
24 TB=20.*ALOG10(T(J))-39.5                                       J   82
   LT=KT+INT(TB)                                                  J   83
25 UA=ABS(U(J))                                                   J   84
   IF (UA-1.06) 26,27,27                                          J   85
26 LU=KU                                                          J   86
   GO TO 30                                                       J   87
27 UB=10.*ALOG10(UA)+0.5                                          J   88
   L=INT(UB)                                                      J   89
   IF (U(J)) 28,28,29                                             J   90
28 LU=KU-L                                                        J   91
   GO TO 30                                                       J   92
29 LU=KU+L                                                        J   93
30 IF (ANE(J)-1.25) 31,31,32                                      J   94
31 LN=KN                                                          J   95
   GO TO 33                                                       J   96
32 ANB=5.*ALOG10(ANE(J))+0.5                                      J   97
   LN=KN+INT(ANB)                                                 J   98
33 IF (TE(J)-100.) 34,35,35                                       J   99
34 LE=KT                                                          J  100
   GO TO 36                                                       J  101
35 EB=20.*ALOG10(TE(J))-39.5                                      J  102
   LE=KT+INT(EB)                                                  J  103
36 IF (LP-LT) 38,37,38                                            J  104
37 LT=LT+1                                                        J  105
38 IF (LU-LP) 42,39,42                                            J  106
39 IF (LU+1-LT) 41,40,41                                          J  107
40 LU=LU-1                                                        J  108
```

```
      GO TO 46                                                          J  109
   41 LU=LU+1                                                           J  110
      GO TO 46                                                          J  111
   42 IF (LU-LT) 46,43,46                                               J  112
   43 IF (LU-1-LP) 44,45,44                                             J  113
   44 LU=LU-1                                                           J  114
      GO TO 46                                                          J  115
   45 LU=LU+1                                                           J  116
   46 LU=MAX0(2,LU)                                                     J  117
      LP=MAX0(2,LP)                                                     J  118
      LT=MAX0(2,LT)                                                     J  119
      LN=MAX0(2,LN)                                                     J  120
      LE=MAX0(2,LE)                                                     J  121
      LU=MIN0(LU,120)                                                   J  122
      LP=MIN0(LP,120)                                                   J  123
      LT=MIN0(LT,120)                                                   J  124
      LN=MIN0(LN,120)                                                   J  125
      LE=MIN0(LE,120)                                                   J  126
      AE=A(LE)                                                          J  127
      AP=A(LP)                                                          J  128
      AT=A(LT)                                                          J  129
      AU=A(LU)                                                          J  130
      AN=A(LN)                                                          J  131
      A(LE)=SME                                                         J  132
      A(LP)=SMP                                                         J  133
      A(LT)=SMT                                                         J  134
      A(LU)=SMU                                                         J  135
      A(LN)=SMN                                                         J  136
      RJ=AMICRN*R(J)                                                    J  137
      IF (RJ-100000.) 48,47,47                                          J  138
   47 RJ=RJ-100000.                                                     J  139
   48 PRINT 61, J,(A(I),I=1,121),RJ                                     J  140
      IF (JSIX-JL) 51,49,51                                             J  141
   49 JSIX=JSIX+6                                                       J  142
      DO 50 K=1,111,10                                                  J  143
      A(K)=DASH                                                         J  144
```

```
      DO 50 L=1,9                                                        J  145
   50 A(K+L)=BLANK                                                       J  146
      A(121)=DASH                                                        J  147
      GO TO 54                                                           J  148
   51 IF (J-1) 54,54,52                                                  J  149
   52 IF (J-JOR) 53,53,54                                                J  150
   53 A(LP)=AP                                                           J  151
      A(LT)=AT                                                           J  152
      A(LU)=AU                                                           J  153
      A(LN)=AN                                                           J  154
      A(LE)=AE                                                           J  155
   54 J=J+JIN                                                            J  156
      PRINT 63, SMP                                                      J  157
      IF (IPRIN-1) 55,56,55                                              J  158
C     PRINT TABLE OF VALUES **                                           J  159
   55 PRINT 59, TIME,DTNHS,N,NE,(IDT(I),I=1,12)                          J  160
      PRINT 60                                                           J  161
      PRINT 62, (J,R(J),P(J),PE(J),U(J),D(J),E(J),T(J),TE(J),EE(J),      J  162
     .       ANE(J),FM(J),J,J=JMIN,JMAX,JIN)                             J  163
   56 CONTINUE                                                           J  164
      IF (NTAPE) 57,58,57                                                J  165
   57 WRITE (8,64) DTN,DTNH,(PH(J),GAMD(J),CHI(J),BET(J),CI(J),ZM(J),    J  166
     .       ZMHI(J),RNH(J),RA(J),RANH(J),J=1,JORP)                      J  167
   58 REWIND 9                                                           J  168
      RETURN                                                             J  169
   59 FORMAT (7H1TIME =,E10.4,2X,6H DTS =,E10.4,2X,4H N =,I4,3X,5H NE =, J  170
     .       I3,3X,12A5,/)                                               J  171
   60 FORMAT (2X,2H J,5X,2H R,9X,2H P,8X,3H PE,9X,2H U,9X,2H D,9X,2H E,  J  172
     .       9X,2H T,8X,3H TE,8X,3H EE,8X,4H ANE,7X,3H FM,8X,2H J,/)     J  173
   61 FORMAT (I4,121A1,F7.1)                                             J  174
   62 FORMAT (I4,11E11.4,I6)                                             J  175
   63 FORMAT (3X,2H 6,8X,2H 5,8X,2H 4,8X,2H 3,8X,2H 2,8X,2H 1,8X,2H 0,   J  176
     .       8X,2H 1,8X,2H 2,8X,2H 3,8X,2H 4,8X,2H 5,8X,2H 6,5H LOG(,A1, J  177
     .       1H))                                                        J  178
   64 FORMAT (8A10)                                                      J  179
      END                                                                J  180
```

```
      SUBROUTINE ZERO                                                   K     1
      COMMON /RVAR/ R(6656)                                             K     2
      COMMON /GJHJ/ A(1536)                                             K     3
      COMMON /RMAS/ Z(2560)                                             K     4
      COMMON /MPROP/ PH(2560)                                           K     5
      COMMON /FREQ/ C(1024)                                             K     6
      COMMON /EQSF/ E(1024)                                             K     7
      DO 1 J=1,6656                                                     K     8
    1 R(J)=0.                                                           K     9
      DO 2 J=1,1536                                                     K    10
    2 A(J)=0.                                                           K    11
      DO 3 J=1,2560                                                     K    12
      PH(J)=0.                                                          K    13
    3 Z(J)=0.                                                           K    14
      DO 4 J=1,1024                                                     K    15
      C(J)=0.                                                           K    16
    4 E(J)=0.                                                           K    17
      RETURN                                                            K    18
      END                                                               K    19
```

APPENDIX II

AIR BREAKDOWN REDUCTION BY TRANSPARENT PARTICLES

The experimentally observed fact that particles of NaCl cause a reduction in the air breakdown threshold by an amount comparable to that of much more opaque materials is difficult to reconcile with a postulated mechanism that requires heating of the particle as an essential step since the heating rate in nearly transparent NaCl is relatively very slow. A lowering of the breakdown threshold occurs when the population of free electrons having kinetic energies above the nitrogen vibrational excitation region is large enough to ensure at least one or more of these is in the laser focal volume to initiate the cascade process. The mechanisms by which electrons can more easily overcome the barrier to energy growth presented by the nitrogen vibrational excitation region usually depend on an air temperature rise induced by the laser heated particle as a result of conduction, shock compression or radiative transfer. Such a temperature rise increases the free electron density and with it the probability an electron will overcome the barrier. In addition, it makes it easier for electrons to thermalize over the barrier, and shifts the balance of energy loss to gain rates in the direction favoring easier breakdown. Without heating, however, these mechanisms are inoperative.

Another mechanism which could allow electrons to overcome this energy barrier without heating could be the result of the presence of a relatively high density of very low-energy electrons in at least a small region close to the surface of the particle. These electrons might be the result of thermionic emission at the particle surface, perhaps consisting of a cloud extending outward no more than the order of a Debye length. Whatever the source of these relatively numerous low-energy electrons, their presence could allow the energy barrier to be overcome as a result of electron-electron collisions alone without the necessity of air heating.

In order to investigate what magnitude of an effect electron-electron collisions might have on easing the penetration of the vibrational excitation barrier, it was necessary to compute electron energy distribution curves with and without electron-electron collisions, and to determine from these whether or not equivalent densities of electrons with energies above the barrier could occur at lower laser powers when electron-electron scattering was taken into account. Although a method for integrating the Boltzman equation directly taking into account electron-electron scattering was available to us (Ref. 40), it required considerable amounts of computer time per case to use. Instead a Monte Carlo method was chosen as a relatively direct approach employing a minimum of simplifying assumptions.

It turned out that, although this direct approach is reasonably effective in treating the case with no electron-electron scattering, it is quite inefficient for the electron-electron scattering case unless a number of more sophisticated Monte Carlo refinements are employed. Resources available to us in this study did not allow completion of incorporation of these refinements, and as a consequence this covers only the work which was done along with an indication of the direction further effort might take.

At the time the Monte Carlo approach was adapted, electron distribution curves had been computed by AFWL (Ref. 30) for the case of no electron-electron scattering utilizing a differencing technique. This technique was subsequently refined by AFWL to include electron-electron scattering effects (Ref. 16). It was hoped that by making comparisons the importance of some simplifying assumptions made in the AFWL approach could be evaluated since they were not made in the Monte Carlo approach. In particular, the differencing approach employed an average power input from the laser, and did not take into account the fact that electrons oscillate in the laser electric field only in the field direction. The kinetic energy of a 2 ev electron in a 10^9 watt/cm^2 laser field may actually range from 1.6 to 2.4 ev during a laser cycle when its initial velocity is aligned with the field. Interaction cross sections can vary significantly over this 0.8 ev energy range,

and this in turn could significantly influence the effectiveness of barrier penetration, and thus the shape of the electron energy distribution curves.

1. MONTE CARLO CALCULATION OF ELECTRON ENERGY DISTRIBUTIONS

In the application of the Monte Carlo technique, the straightforward approach of following the time history of a single electron starting at some initial kinetic energy was chosen. This electron was followed until it was lost by some process such as attachment, or until it caused an ionization event. After an ionization event, one of the resultant electrons was picked to follow further. The total time spent by the electron at each energy (in practice the time spent in one of the preselected energy intervals) was recorded. From this information, the energy distribution function was calculated.

The ergodic hypothesis assures the validity of this method of determining energy distributions for steady state distributions. It states that the time average for the properties of any system of interest equal the averages of those same properties for the members of the ensemble (Ref. 41). That is, any quantity, A(p, q) will have a time average,

$$\overline{A} = \lim_{T \to \infty} \frac{1}{T} \int_0^T A(p(t),\ q(t))\, dt \qquad (177)$$

and an ensemble average

$$< A > = \int A\ (p, q)\ f(p, q)\ dp\ dq \qquad (178)$$

where f(p, q) ≡ the distribution function

$p(t) \equiv p\ (p_o,\ q_o,\ t)$, momentum coordinates

$q(t) \equiv q\ (p_o,\ q_o,\ t)$, position coordinates

the ergodic hypothesis guarantees the ultimate equality of these two average values, $\bar{A}$ and $< A >$ at sufficiently long times.

When the distribution is changing in time, these two average values are equal only if the time variation is slow compared with the time for the distribution function to relax to its quasi-equilibrium form. It will be assumed that this is true for the cases of interest here, although this assumption needs further verification at least in the late stages of breakdown, when electron-electron collisions, superelastic collisions, etc., are becoming increasingly important at a rapid rate.

If the quantity, A, is taken to be the electron kinetic energy, E, and if further the integrals are replaced with summations, these results are

$$\bar{E} = \lim_{T \to \infty} \sum_i E_i \left(\frac{\Delta t_i}{T} \right) \tag{179}$$

and

$$< E > = \sum E_i f(E_i) \tag{180}$$

where Δt_i is the time spent by the electron in the ith energy bin represented by energy, E_i. The ergodic hypothesis states that $\bar{E} = < E >$ for all ensembles. For this to be true for all arbitrary E_i requires from equations (179) and (180) that

$$f(E_i) = \lim_{T \to \infty} \left(\frac{\Delta t_i}{T} \right) \tag{181}$$

Equation (181) proves that the results of following one electron's history, as described above, gives asymptotically the energy distribution function desired. The statistical uncertainties in the distribution at any finite time are equivalent to those inherent in the corresponding Monte Carlo calculation.

In following the history of a single electron, its interactions with molecules were simulated. The reactions with air molecules considered were excitation of vibrational and ground to excited electronic states as well as elastic collisions. Oxygen attachment was also included. In the calculations done to date only the ground states were considered to be populated significantly, although no basic difficulty would be encountered in including excited states. Emperical cross-section data for the various interaction channels were obtained from the literature (Refs. 4, 5, 6, 7, 42 and 43). The specific interaction channel was chosen randomly in proportion to the corresponding cross section, and for the channel chosen the electron kinetic energy was reduced appropriately upon collision. A new random direction for the test electron velocity vector was picked after the collision on the assumption of isotropy.

a. Interaction of Electron with Laser Electric Field

The response of the electron to the oscillating electric field of the laser beam was treated classically. By the correspondence principal, this treatment is correct in the limit where the electron kinetic energy gained is large compared to a laser photon energy. The inverse bremsstrahlung absorption of energy by the electron upon scattering from another eiectron or the scattering of an electron from an ion charge center is correctly described classically in the many photon energy limit for the same reason. It has been shown that when the energy imparted by such a coulomb scattering is small compared to the photon energy, a classical treatment of the scattering also gives the correct answer, where energies of only a fraction of a single photon energy are interpreted as probabilities of there being a one photon energy transfer (Ref. 4). In the intermediate region multiple photon interactions become important and are hard to evaluate. It is assumed that the classical treatment is adequate in this range also.

During a single laser light cycle, the electrons oscillate while the neutral particles remain essentially stationary. Ions can be treated as stationary because of their relatively large mass. The temperature of interest are

low enough so that all particles except electrons can be assumed to have negligible translational velocity at all times. All electron energy distribution curves presented are specified only after removal of the kinetic energy of oscillation due to the electric field, or effectively when the field is turned off, to avoid ambiguity. A collision while the field is on randomizes the directed component of the electron velocity resulting from the oscillating field and thus thermalizes this part of the electron kinetic energy. The velocity of impact of an electron with another particle varies during a laser light cycle, and with it the impact energy and cross section for interaction.

From a classical point of view, the electric vector of the light in a polarized laser beam can be represented by

$$E = E_0 \cos(\omega t) \tag{182}$$

where t is time, and the frequency is given by

$$\omega = 2\pi c/\lambda \tag{183}$$

with c the velocity of light and λ the light wavelength. The peak electric field, E_0, in volts/cm is related to the average power in the laser beam, P, in watts/cm^2 by

$$E_0^2 = 240\pi P \tag{184}$$

A free electron in this field has an induced velocity component given by

$$v_E = V_0 \sin(\omega t) \tag{185}$$

where

$$V_0 = \frac{eE_0}{\omega m_0} \tag{186}$$

and e the charge and m_0 the mass of an electron.

Combining equations (183) through (186) gives the numerical formula

$$V_0 = 25.66\ \lambda\sqrt{P} \tag{187}$$

where V_0 is in cm/sec, λ in microns, and P in watts/cm^2.

The actual electron velocity is the vector sum of that due to thermal motion if the laser were off, $\vec{v}_0$, and that caused by the field:

$$\vec{v} = \vec{v}_0 + \vec{v}_E \tag{188}$$

Neglecting the motion of all gas particles due to either temperature or laser field, the electron kinetic energy of impact with other gas particles is given simply by

$$W = \tfrac{1}{2} m_0 v^2 \tag{189}$$

where v is obtained from equation (188).

Using equations (187, 188 and 189), it can be shown that the maximum range of variation in kinetic energy of an electron oscillating in the laser field is

$$\Delta E = 0.6\ \sqrt{P \times W} \tag{190}$$

where ΔE is in electron volts, P is in units of 10^9 watts/cm^2 and W is in ev.

b. Interaction of the Electron with another Particle

In an increment of time Δt, the electron may interact with a gas particle or another electron or not at all. If the time increment is chosen small enough so that during it the laser-induced velocity increment is negligible, then the electron kinetic energy may be taken as constant during this

time increment. For a given electron energy, the cross sections all-particle interaction channels are known. The probability the electron will survive a time, t, without collisions is given by the Poisson probability

$$p(t) = e^{-tv\Sigma\sigma_i n_i} \tag{191}$$

where the cross sections are σ_i for a number/volume, n_i, of interaction targets of type i.

In the increment of time, Δt, the probability of a collision is from equation (191)

$$\Delta p = 1 - e^{-\Delta tv\Sigma\sigma_i n_i} \tag{192}$$

If the time interval is chosen so

$$\Delta tv\Sigma\sigma_i n_i << 1 \tag{193}$$

then equation (192) can be approximated as

$$\Delta p \cong \Delta tv\Sigma\sigma_i n_i \tag{194}$$

Equation (194) may be used to choose an interaction channel in the Monte Carlo calculation as follows. A random number, R, is chosen between 0 and 1.

$$0 < R < 1 \tag{195}$$

The quantity

$$q_m = \Delta tv \sum_{i=1}^{m} \sigma_i n_i \tag{196}$$

is computed for successive values of m and for each value q_m is compared with R. If for any m it is found that

$$q_m > R \tag{197}$$

then the electron is chosen to interact in channel m during Δt. If after summing over all channels the inequality of equation (197) is not satisfied, the electron is chosen to not interact in this particular time interval. For this method to be valid, it is necessary that the time step, Δt, meet condition (193).

The electron-electron scattering cross section must be included in evaluating the condition (193). For this purpose, it will be assumed no interaction occurs beyond the Debye shielding distance given by (Ref. 44)

$$h = \sqrt{\frac{kT}{4\pi m_e e^2}} = K\sqrt{\frac{T}{n_e}} \tag{198}$$

where n_e is the volume density of electrons, and e the electronic charge. For T in deg kelvin and n_e in cm^{-3}, K = 6.90 gives h in cm. The cross section for electron-electron interaction is thus

$$\sigma_e = \pi h^2 = \pi K^2 T/n_e \tag{199}$$

It is noteworthy that in the summation of equations (193) and (194) the product $\sigma_i n_i$ for electrons from equation (199) is

$$\sigma_e n_e = \pi K^2 T \tag{200}$$

which is independent of electron density. Since the temperature, T, will not generally correspond to a Maxwellian distribution, in its place $\frac{2}{3}$ times the

average kinetic energy of the electron distribution was used.

If the restriction on-time interval imposed by equation (193) is much more severe than imposed by laser frequency, then it can be more efficient computationally to relax this condition by determining first whether there is any interaction at all by using the condition for there to be one that Δp from equation (194) exceed a random number, R. Only after the existence of interaction is established is a channel for it chosen.

c. Electron-Electron Scattering

In the event of an electron-electron scattering, a target electron must be picked from the distribution for the test electron to scatter from. The second electron energy cannot be simply selected in proportion to its frequency of occurrence in the distribution, because the thermal velocity is not small compared to that of the test electron.

The probability per unit time that the test electron with velocity, $\vec{v}_1$, will pass within a Debye length of a target electron is given by

$$\frac{dP}{dt}\left(\vec{v}_1, \vec{v}_2\right) = \pi h^2 \int_0^{2\pi} d\theta \int_0^{\pi} d\varphi \sin\varphi n'(v_2, \theta, \varphi) \left|\vec{v}_1 - \vec{v}_2\right| \qquad (201)$$

where $n'(v_2, \theta, \varphi)$ is the density of target electrons with velocity $\vec{v}_2$ and speed v_2 in a spherical coordinate system with polar angle, φ, and azimuth angle, θ. It will be assumed that the target electrons are isotropically distributed in velocity direction, or

$$n'(v_2, \theta, \varphi) = n(v_2)/4\pi \qquad (202)$$

Consequently, equation (201) using (202) becomes

$$\frac{dP}{dt}\left(\vec{v}_1, v_2\right) = \frac{n(v_2)\,\pi h^2}{2} \int_0^{\pi} \sqrt{v_1^2 + v_2^2 - 2v_1v_2\cos\varphi}\ \sin\varphi\, d\varphi \qquad (203)$$

which gives taking into account the absolute value of the velocity difference in equation (201)

$$\frac{dP}{dt}\left(\vec{v}_1, v_2\right) = \frac{n(v_2)\,\pi h^2}{3} \left(3v_A + V_B^2/V_A\right) \qquad (204)$$

where

$$v_A = \max\,(v_1, v_2)$$

$$v_B = \min\,(v_1, v_2)$$

The quantity $n\,(v_2)$ in equation (204) may be determined from the distribution function for target electron energies. Thus, for a given test electron velocity the modified distribution function implied by equation (204) may be used to correctly select a target electron speed, v_2.

The direction of this target electron is assumed isotropically distributed. However, even though it is now determined that an interaction of the test electron will occur with a target of speed, v_2, the probability the collision will occur depends upon the relative velocity of the two electrons. The velocity of the target electron may be randomly selected by first selecting a random number $0 < R < 1$. The cosine of the angle between the two electron velocities is then given by

$$\mu = \frac{v_1^2 + v_2^2 - \left[\left(1 - R\right)\left(v_1+v_2\right)^3 + R\,\left|v_1 - v_2\right|^3\right]^{\frac{2}{3}}}{v_1v_2} \qquad (205)$$

The vector velocity of the target electron is then

$$\vec{v}_2 = v_2 \left[\left(\mu - \gamma\beta \frac{\vec{v}_1}{v_1} + \beta \hat{R} \right) \right] \tag{206}$$

where $\hat{R}$ is a random unit vector and

$$\gamma \equiv \frac{\vec{v}_1 \cdot \hat{R}}{v_1} \neq 1$$

and $$\beta \equiv \sqrt{\frac{1 - \mu^2}{1 - \gamma^2}}$$

The two electrons are now allowed to interact in a center of mass encounter and the new test electron energy and corresponding velocity having a random azimuth angle found and converted into the laboratory frame.

The velocities in the electron-electron center of mass system are

$$\vec{w}_1 = (\vec{v}_1 - \vec{v}_2)/2 = \vec{w}$$
$$\vec{w}_2 = -\vec{w} \tag{207}$$

The angle of scattering in the cm system is found by picking a random number $0 < R < 1$, and finding the impact parameter, x, in terms of the Debye length, h, from

$$\pi x^2 = R\pi h^2 \tag{208}$$

and

$$x = h\sqrt{R}$$

The collision diameter is given by

$$b = e^2/(m_0 W^2) = \left(\frac{e^2}{mc^2}\right)\left(\frac{c}{w}\right)^2 \tag{209}$$

The scattering angle, θ, is found from

$$\theta = 2 \tan^{-1}\left(\frac{b}{2x}\right) \tag{210}$$

Equation (210) using equation (209) gives

$$\cos\theta = \frac{4h^2R - b^2}{4h^2R + b^2} \tag{211}$$

in terms of the random number, R. The azimuth angle is still undetermined and can be picked at random as follows. Let $\hat{R}$ be a randomly picked unit vector. If $\hat{w}$ is a unit vector in the direction of $\vec{w}$, then the center of mass velocity, $\vec{w}'$, after scattering is given by

$$\vec{w}' = \left[\frac{\sin\theta\hat{R} + }{\sqrt{1 - (\hat{W}\cdot\hat{R})^2}}(\cos\theta) - \frac{\sin\theta(\hat{w}\cdot\hat{R})\hat{w}}{\sqrt{1 - (W\cdot R)^2}}\right] \tag{212}$$

Shifting back to the laboratory system, the velocity of the test electron after scattering is

$$\vec{v}_1' = \frac{(\vec{v}_1 + \vec{v}_2)}{2} + \vec{w}' \tag{213}$$

From this velocity the electron kinetic energy after the electron-electron scattering even can be found using equation (189).

A computer program, Laser Ionization Rates by Single Electron Monte Carlo (LIRSEM), was written to carry out the calculations described above. It is described in Appendix III. It was used to compute electron energy distribution curves without electron-electron scattering. The results are shown for two laser powers for which calculations were also made by AFWL along with the AFWL results for comparison in figures 23 and 24. Results obtained for a number of other laser powers are shown in figure 25.

It can be seen in the comparison figures that the curves are similar in shape above a few electron volts in energy, but differ considerably at lower energies. There are three plausable reasons for the low-energy differences. One is that the energy bins used in the Monte Carlo (0.2 ev) were too coarse to provide adequate resolution at very low energies. Another is that the Monte Carlo maximum energy was at 20 ev. Any electron exceeding this energy was considered to have caused an ionization to occur. This could artificially lower the high-energy tail of the Monte Carlo curves and at the same time return too many electrons to the minimum energy bin thus distorting the distribution. A third possible reason is that the distributions really are different as a result of the approximations made in the AFWL calculations which are absent in the Monte Carlo.

Monte Carlo calculations were also made including the electron-electron scattering feature of the computer program. However, an error was subsequently found in the program. This resulted from assigning a random direction to the target electron after its speed had been found rather than using equations (205) and (206) to determine its velocity. Consequently, these results are in error, and are not reported here. The program described in Appendix III has not been corrected, but could be relatively easily for this omission. However, another more grave difficulty was also discovered which will take considerably more effort to correct. This difficulty concerns the efficiency of the Monte Carlo method when electron-electron scattering is included.

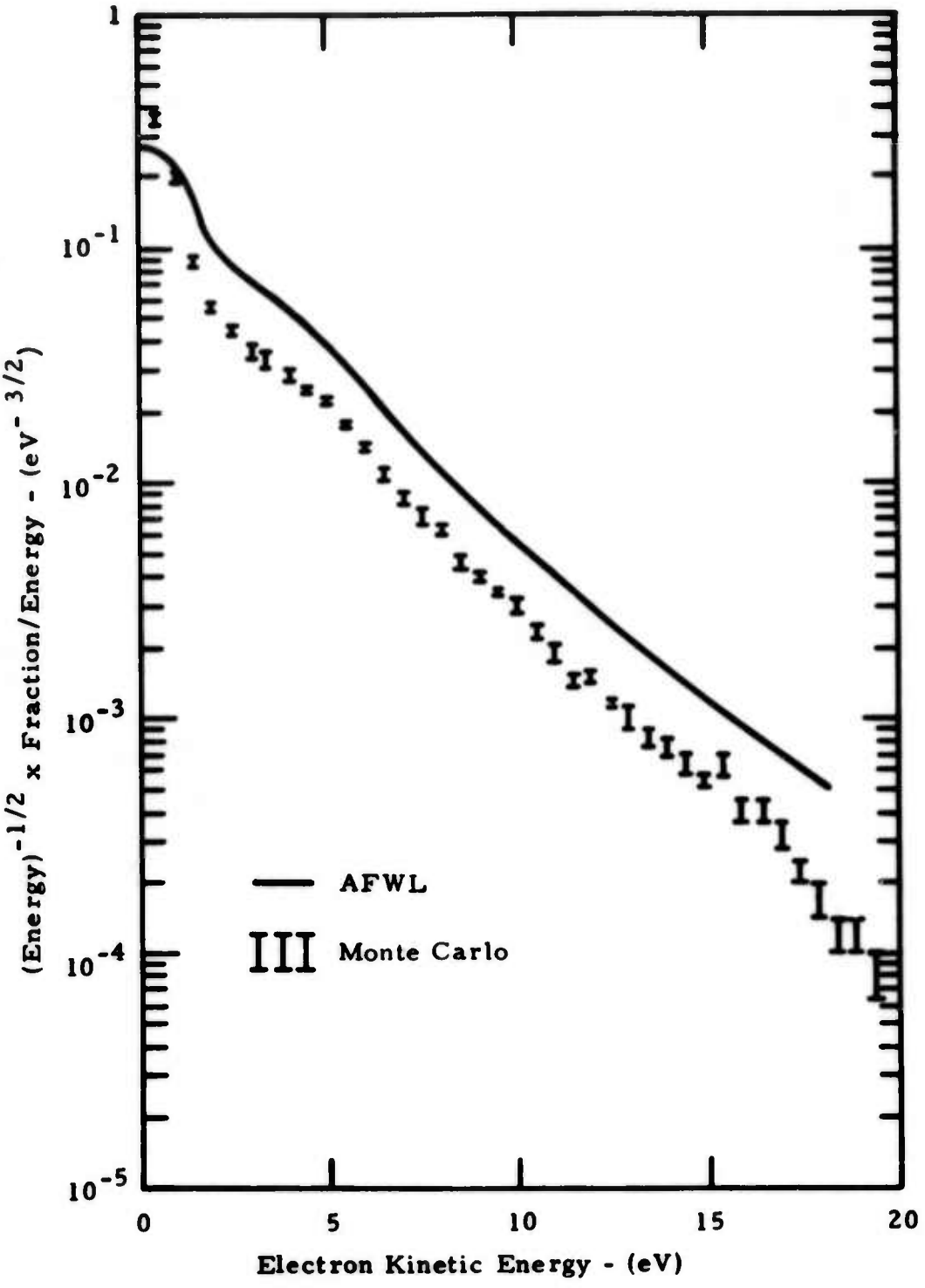


Figure 23. Comparison of electron energy distributions for laser power of 8×10^9 watts/cm^2

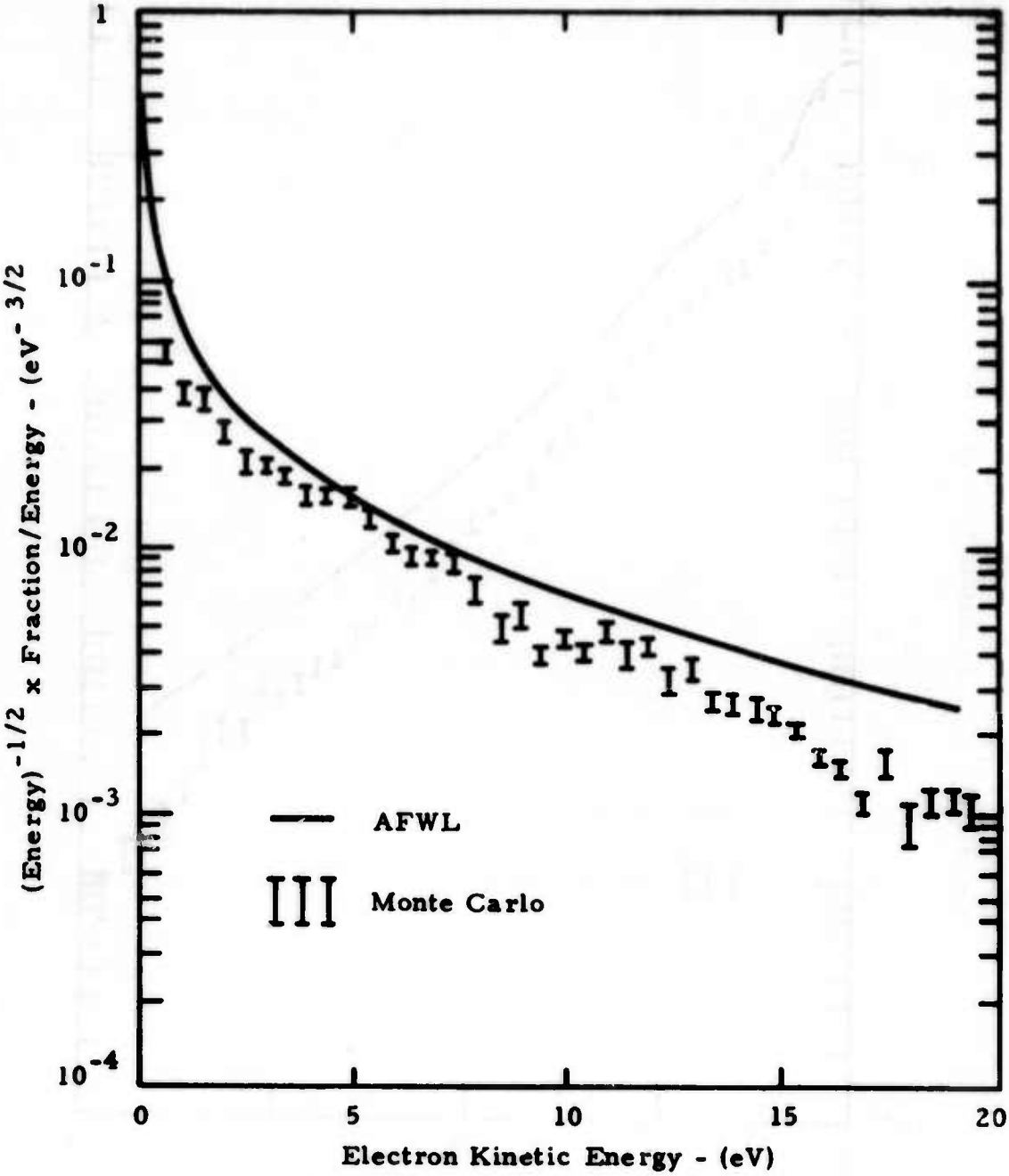


Figure 24. Comparison of electron energy distributions for laser power of 2×10^{11} watts/cm^2

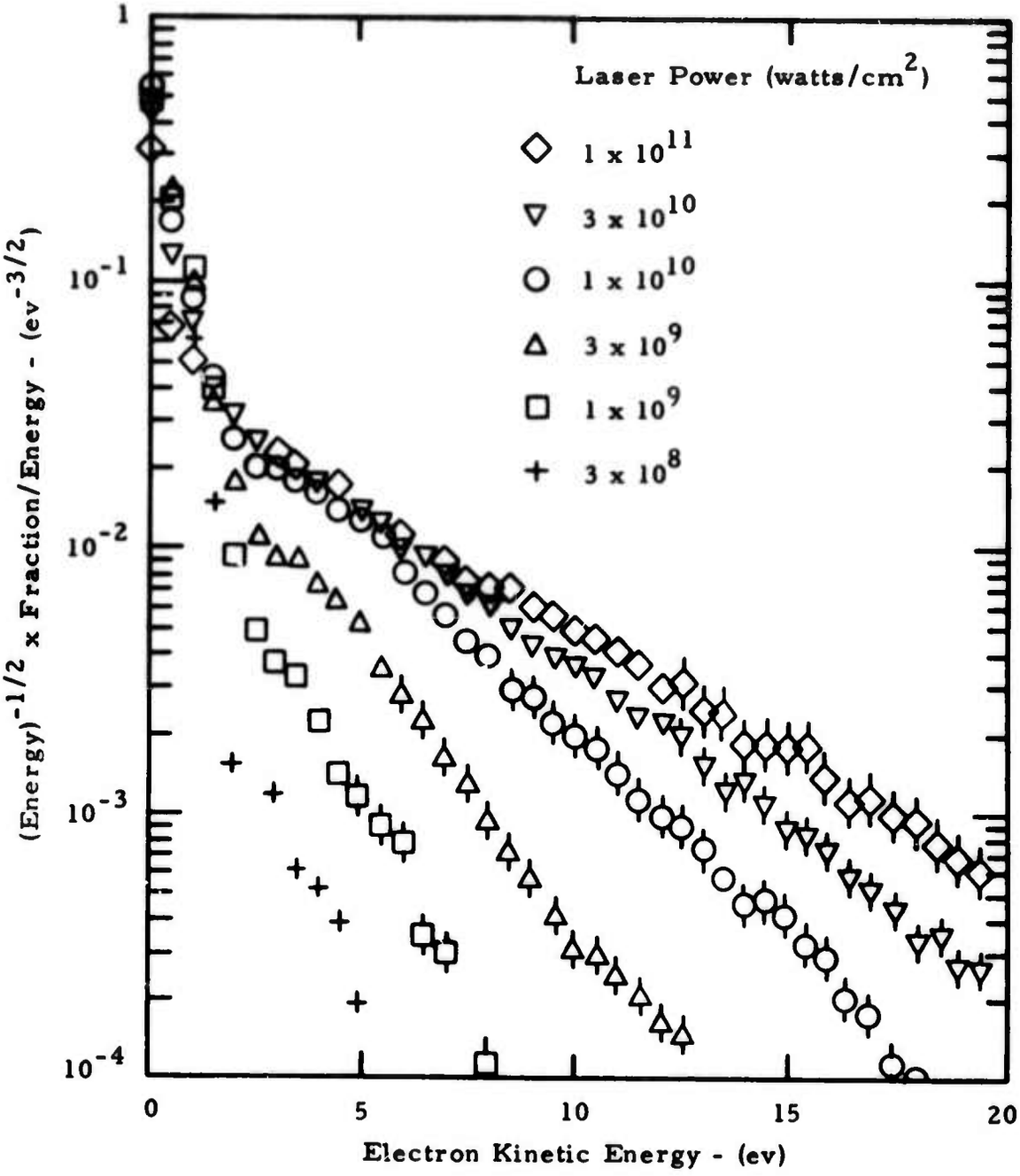


Figure 25. Electron energy distribution functions ($f/E^{1/2}$) from Monte Carlo calculations for various CO_2 laser powers.

The Monte Carlo method employed in LIRSEM is categorized as the "crude" method in a treatise on Monte Carlo methods (Ref. 45). In this method, the statistical uncertainty assigned to the magnitudes of the times spent in each energy bin of the distribution are given by the magnitude and the time spent divided by the square root of the number of entries into the bin when electron-electron scattering is neglected. When electron-electron scattering is included, however, the fact that a target electron was picked by a second random process increases the uncertainty by the square root of the reciprocal of the fraction of the time spent in the bin. In the first instance acceptable accuracy is obtained quite quickly, but in the second only very slowly. Although time did not permit incorporating more efficient methods, a number exist. Either a form of importance sampling using a Maxwellian distribution as a reference function, or one of the sampling or splitting methods would be appropriate (Ref. 45). The rate of attaining a desired accuracy can be increased by orders of magnitude when these better methods are employed which take maximum advantage of all that is known about the answer ahead of time, and which employ Monte Carlo only were absolutely necessary.

With the corrections, improvements, and refinements suggested, it is felt that the Monte Carlo approach to determining electron distribution functions can be made to be a valuable approach. As more emperical cross-section data are introduced into the analysis and as more simplifying assumptions are discarded, the relative value of the Monte Carlo approach increases until for sufficiently complex phenomenon it may become the only feasible approach.

APPENDIX III

MONTE CARLO COMPUTER PROGRAM

A computer program, LIRSEM, was written to carry out the calculations described in Appendix II. A logical flow chart for this program and a listing are included.

The program is dimensioned to allow for 100 energy bins for the electrons. As run, the bins are 0.2 ev wide ranging from 0 to 20 ev energies. Cross-sections are first computed for each interaction channel limited to 30 by present dimensioning. Cross-sections are read into the program as data in the form of energy cross section pairs of points arbitrarily spaced from which mesh values are interpolated. After reading the cross-section data, an initial electron distribution is read in the same format after first reading an arbitrary heading card.

The format for this input data is the following:

Card 1. NCHAN, Number of channels, N, (≤ 30) in columns 1-5 in 1I5 format.

This is followed by N set of cards consisting of the following:

Card 2. ICHAN, Channel number, J, in 1-5 in 1I5 format
NJCHAN, No. of pairs of data points in columns 6-10, 1I5 format
ITYPE(J), Type of cross section according to the code (in columns 11-15 in 1I5 format)

1 = total cross section
2 = ionization cross section
3 = vibrational excitation cross section
4 = attachment cross section

PARTCL(J), Number density (cm^{-3}) of target particles in this channel in columns 21-30 in E10.4 format

ELOSS(J), Energy loss upon collision (ev) when appropriate for this type of interaction in columns 31-40 in E10.4 format.

Card 3 or more.

Pairs of energy (ev), cross section ($1 \times 10^{-16} cm^2$) data points in 16F5.2 format.

After reading N sets of cross-section data, an arbitrary heading card in 80A1 format is read followed by one more set of cards in the Card 2 and 3 formats. In this instance, the first word on the first card serves only to indicate the run is to continue if it is nonzero. The next cards in the same form as Card 3 above represent energy (ev), relative distribution height (arbitrary units) data pairs to specify the initial electron distribution. The normalization is discussed below. A run may be terminated by use of a blank card for Card 2.

Following this, control data is read in unformatted input form (Namelist). The variables read in are

WAVE = laser wavelength (cm)
POWER = laser power ($watts/cm^2$)
ENGZER = initial energy of test electron (ev)
ELECTS = free electron density (cm^{-3}), a constant; program control constants:
NDT = number of time step subdivisions made in the basic laser cycle
NEDIT = the number of basic cycles before an edit is made
NQNIT = the number of basic cycles before run is terminated
NSTART = number of basic cycles assumed to have transpired before input electron distribution occurs. (Serves to normalize this distribution)

NOMOL = ignored if less than or equal to zero. If greater than zero molecular interactions bypassed for pure electron-electron scattering

IWRT = outputs computed cross sections for all channels and bins if greater than zero. If less than one this output bypassed.

If values for these control constants are not specified in the namelist, they are assigned the following values:

NDT = 0
NEDIT = 10,000
NQUIT = 100,000
NSTART = 100
NOMOL = 0
IWRT = 0

Any number of sets of initial electron distributions followed by namelist specifications may be stacked. A blank card terminates the run normally.

In addition to the control constants, a number of interval constants of the program are printed. Also, values of the following variables are printed.

DT = basic time step (sec)
VELZER = maximum velocity induced by laser field (cm/sec)
ANGLE = phase angle (radians) corresponding to one time step
TELECT = average electron temperature (ev)

At each edit time, the electron energy distribution in sec/bin is given as well as the fraction/bin and the number of basic time steps/bin in tabular form. This is followed by a table of values for

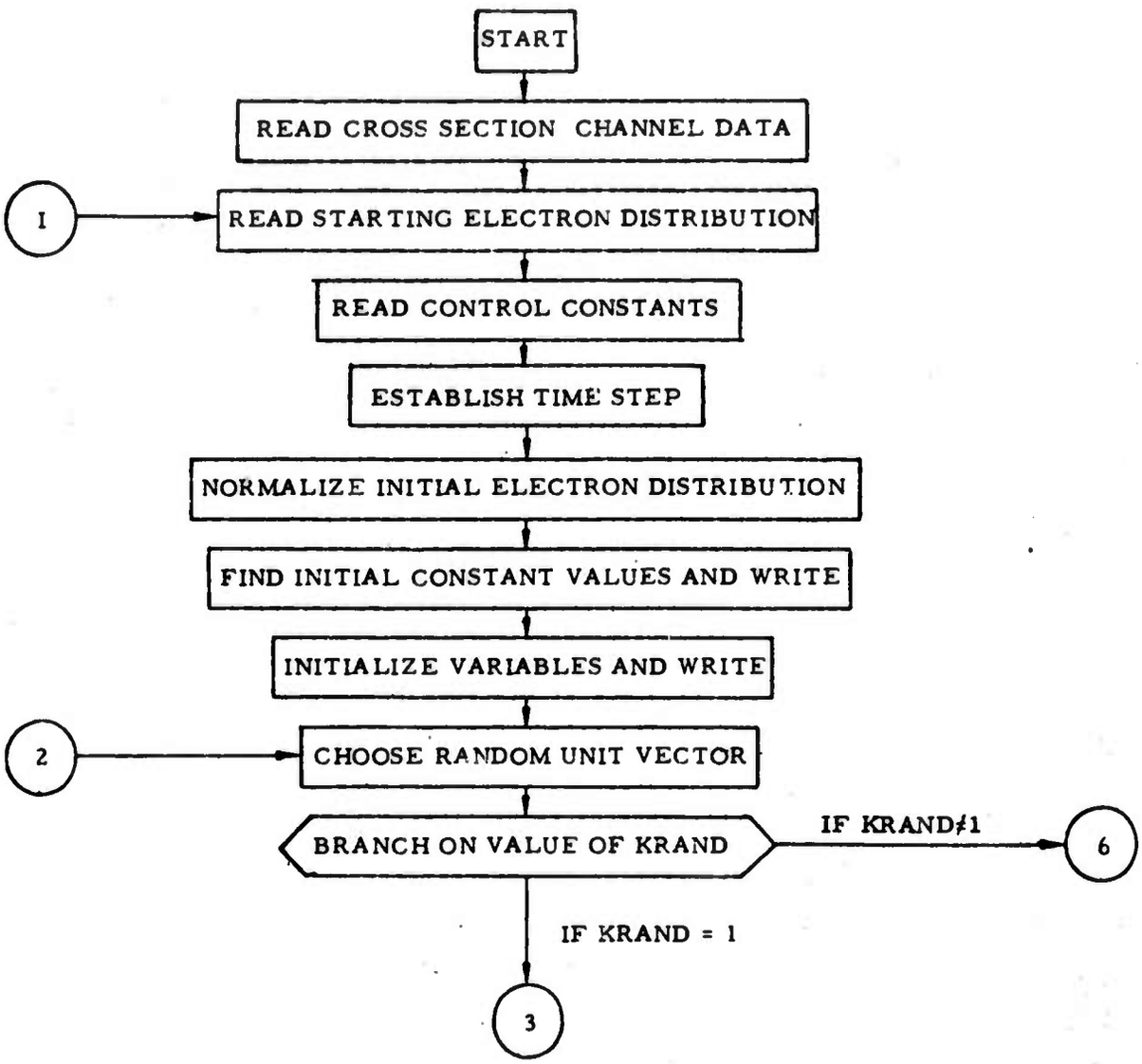
T	=	time of edit (sec)
RATION	=	ionization rate in ionizations per second per cc
RATERR	=	statistical uncertainty in RATION
NION	=	number of ionization events
NCYION	=	cycle number of last ionization
NCY	=	total cycles at edit
NEE	=	number of electron-electron collisions
NEM	=	number of electron-molecule collisions
NWRITE	=	cycles to write
TEV	=	temperature of electrons (ev)
EVT	=	kinetic energy of test electron (ev)
VT	=	last electron velocity components (cm/sec) with field on
VZ	=	last electron velocity components (cm/sec) if field were off
NEL	=	number of elastic collision
NELU	=	number of electron-electron scatterings for which the cosine of the scattering angle was less than ELNO(= .99999)
XPBELC	=	log of probability of an electron scattering in one cycle
PBELC	=	probability of an electron scattering in one cycle
LOGIC	=	value (T, F) of variable to decide if electrons
JE	=	number of steps into last laser cycle
NATCH	=	number of attachments
ELNO	=	cosine of electron-electron scattering angle above which scatterings were ignored
VEZ	=	electron velocity (cm/sec) after scattering with field off
JE2	=	last channel chosen

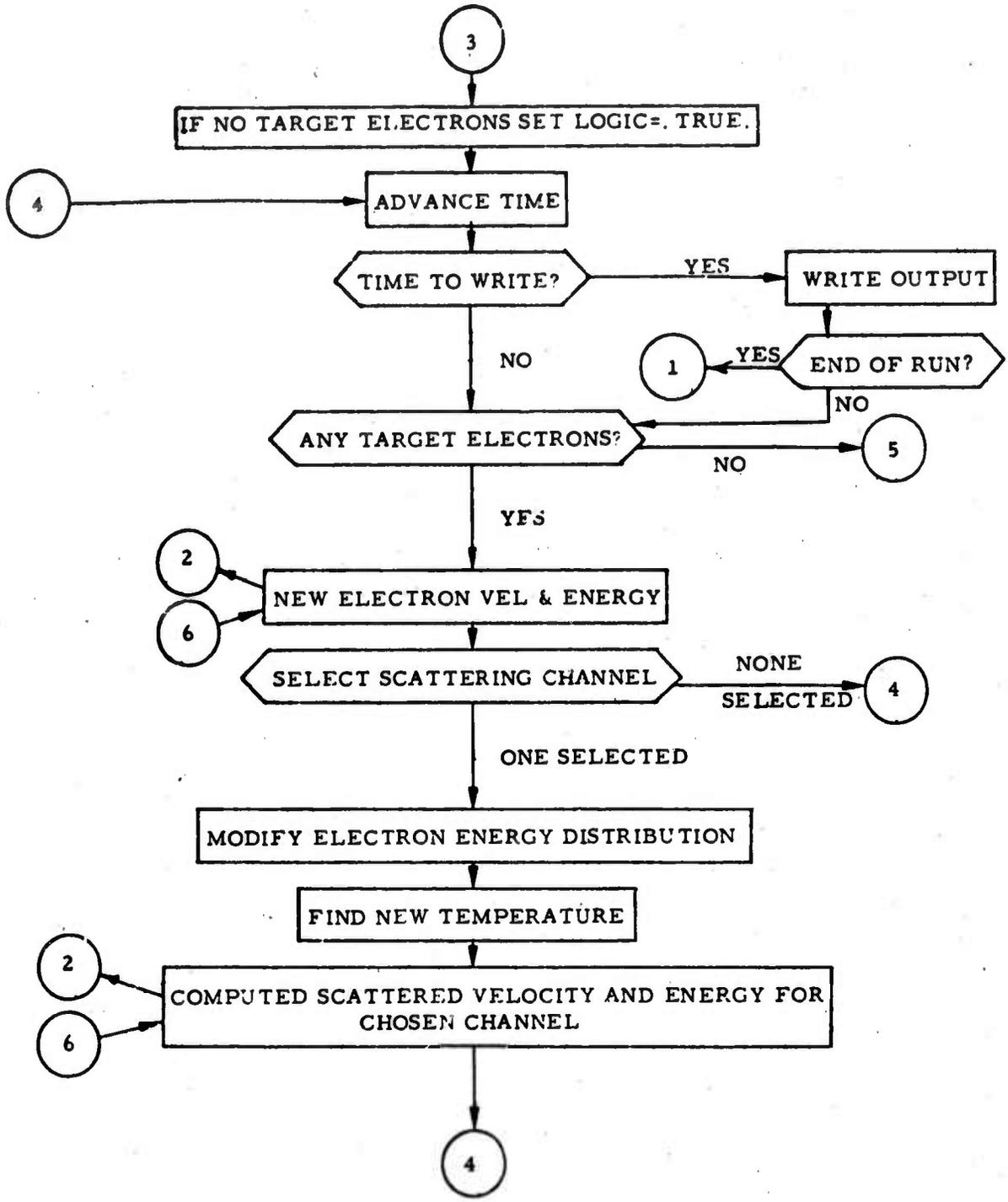
These output values are followed by a plot of the electron energy distribution function at the edit time. The values of the points plotted (in 0.5 ev bins) as well as their uncertainty are listed next to the plot.

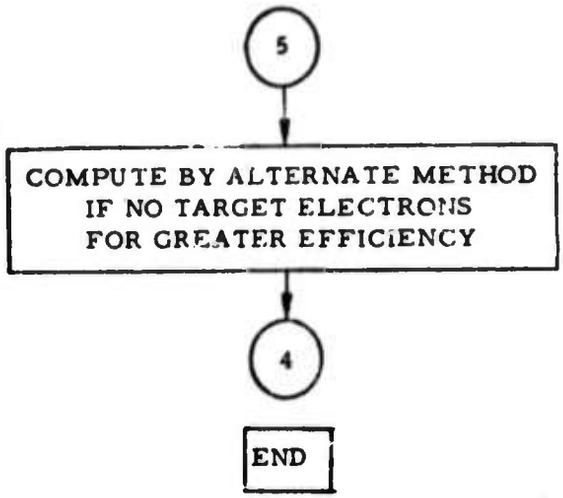
The formula employed in the LIRSEM program follow closely those given in Appendix II. One variation was a faster means of advancing time when there are no electron-electron collisions was devised. Essentially it first randomly selects a laser cycle in which there is an event using the cross section averaged over one cycle in this selection. This section of the program is titled "No-Electron Channel Choice."

Logical Flow Chart for

LIRSEM Computer Program




3
IF NO TARGET ELECTRONS SET LOGIC=. TRUE.
4
ADVANCE TIME
TIME TO WRITE?
YES
WRITE OUTPUT
NO
1
YES
END OF RUN?
NO
ANY TARGET ELECTRONS?
5
NO
YES
2
6
NEW ELECTRON VEL & ENERGY
SELECT SCATTERING CHANNEL
NONE SELECTED
4
ONE SELECTED
MODIFY ELECTRON ENERGY DISTRIBUTION
FIND NEW TEMPERATURE
2
6
COMPUTED SCATTERED VELOCITY AND ENERGY FOR CHOSEN CHANNEL
4


5
COMPUTE BY ALTERNATE METHOD
IF NO TARGET ELECTRONS
FOR GREATER EFFICIENCY
4
END

```
C
C     LASER IONIZATION RATES BY SINGLE ELECTRON MONTE-CARLO
C
C          SPACE RESERVATION
C
      DIMENSION TE(100),SIG(20,100),EIN(25),VAL(25),VE(100),
     1 PARTCL(20),XPTOT(100),PBTOT(100),XPROB(20,100),ELOSS(20),
     2 VT(3),VZ(3),VZ2(3),W(3),IX(45),IY(45),IT(80),ITX(80),
     3 ITY(80),R(3),JEC(200),VF(200),H(10)
      LOGICAL LOGIC
      NAMELIST /CONTRL/ WAVE,POWER,ENGZER,NDT,NEDIT,NQUIT,
     1 NSTART,ELECTS
      NAMELIST /CONSTS/CLGT,CPLV,TWOPI,CETOV,PI,CVTOE,EPCON,
     1 APROX,CAA,CBB,CELOSS
      NAMELIST /OUTPUT/ T,RATION,RATERR,NION,NCYION,NCY,NEE,NEM,NWRITE,
     1 TEV,EVT,VT,VZ,NEL,NELU,XPBELC,PBELC,LOGIC,JE
      NAMELIST /VALUES/DT,VELZER,ANGLE,TELECT
      DATA CLGT/3.E10/CPLV/7.686E7/TWOPI/6.2831852/CETOV/5.9287E7
     1 /PI/3.1415926/CVTOE/2.845E-16/EPCON/1.736E6/APROX/.01
     2 /CAA/5.526E5/CBB/6.4323E16/CELOSS/.9989126/CSQANG/1.E-16/
     3 ELNO/.999/
C
C          FORMATS
C
 1000 FORMAT(1H1,5X,26HINPUT CARDS LISTED AS READ/)
 1010 FORMAT(2I5,2E10.4)
 1011 FORMAT(1X,2I5,2E10.4)
 1020 FORMAT((16F5.2))
 1021 FORMAT(1X,(16F5.2))
 1030 FORMAT((80A1))
 1040 FORMAT(21X,7HTIME = ,1P1E10.3,7H SECOND //)
 1050 FORMAT(1H0,1A6/(1X,1P10E12.3))
C
```

LIRSEM Listing (Cont)

```
C          READ HISTOGRAM CARDS
C
      READ(5,1030) ITX,ITY
C
C          READ IN CROSS SECTIONS
C
C               DATA CARD HEADING
C
   10 WRITE(6,1000)
C
C               CROSS-SECTION POINTS
C
      READ (5,1010) NCHAN
      WRITE(6,1011) NCHAN
      I=0
   15 I=I+1
      IF(I.GT.NCHAN) GO TO 100
      READ (5,1010)ICHAN,NJCHAN,PARTCL(ICHAN),ELOSS(ICHAN)
      WRITE(6,1011)ICHAN,NJCHAN,PARTCL(ICHAN),ELOSS(ICHAN)
      KGO=1
C
C                    READ AND INTERPOLATE
C
   50 READ(5,1020) (EIN(J),VAL(J),J=1,NJCHAN)
      WRITE(6,1021)(EIN(J),VAL(J),J=1,NJCHAN)
      DO 99 J=1,10
      EV=.2*J-.1
      IF(EV-EIN(1)) 90,60,60
   60 DO 80 K=2,NJCHAN
      IF(EV-EIN(K))70,70,80
   70 TE(J)=((EIN(K)-EV)*VAL(K-1)+(EV-EIN(K-1))*VAL(K))/
     1 (EIN(K)-EIN(K-1))
      GO TO 99
   80 CONTINUE
   90 TE(J)=0.
   99 CONTINUE
```

LIRSEM Listing (Cont)

```
      GO TO (20,120),KGO
   20 CONTINUE
      DO 30 J=1,100
   30 SIG(ICHAN,J)=TE(J)*CSQANG
      GO TO 15
C
C          READ HISTOGRAM TITLE CARD
C
  100 READ(5,1030) IT
C
C          READ IN INITIAL ELECTRON DISTRIBUTION
C
  110 READ (5,1010) NJCHAN
      IF(NJCHAN.LT.1) STOP NORMAL
      WRITE(6,1011) NJCHAN
      KGO=2
      GO TO 50
  120 CONTINUE
C
C
C          WRITE CONSTANTS
C               PRESET VALUES
C
      WRITE(6,CONSTS)
      NDT=100
      NEDIT=1000
      NQUIT=100000
      NSTART=100
C
C               CONTROL CONSTANTS
C
      READ(5,CONTRL,ERR=130)
      WRITE(6,CONTRL,ERR=130)
      GO TO 135
  130 STOP CONTRL
  135 IF(NDT.GT.200.OR.NDT.LT.1) STOP CYCLE
```

LIRSEM Listing (Cont)

```
C
C          BASIC DERIVED CONSTANTS
C
C               TIME STEP
C
      DT=WAVE/(CLGT*NDT)
C
C               NORMALIZE ELECTRON DISTRIBUTION
C
      SUM=0.
      DO 140 I=1,100
  140 SUM = SUM + TE(I)
      SUM = NSTART*DT/SUM
      DO 150 I=1,100
  150 TE(I)=SUM*TE(I)
C
C               FIELD INDUCED VELOCITY
C
      VELZER=CPLV*WAVE*SQRT(POWER)
      ANGLE=TWOPI/FLOAT(NDT)
      DO 160 I=1,NDT
  160 VF(I)=VELZER*SIN((I-1)*ANGLE)
C
C               INITIALIZE PROBABILITIES AND TEMPERATURE
C
      SUM=0.
      TEV=0.
      DO 180 J=1,100
      EV=.2*J-.1
      VE(J)=CETOV*SQRT(EV)
      SUM=SUM+TE(J)
      TEV=TEV+EV*TE(J)
      XPTOT(J)=0.
      DO 170 I=1,NCHAN
      XPROB(I,J)=DT*VE(J)*SIG(I,J)*PARTCL(I)
  170 XPTOT(J)=XPTOT(J)+XPROB(I,J)
```

LIRSEM Listing (Cont)

```
  180 PBTOT(J)=EXP(-XPTOT(J))
      TEV=TEV/SUM
      TELFCT=TEV
C
C          WRITE ALL PRECOMPUTED CONSTANTS
C
      L=6HPARTCL
      WRITE(6,1050) L,(PARTCL(I),I=1,20)
      L=6HVF
      WRITE(6,1050) L,(VF(I),I=1,NDT)
      L=6HXPROB
      WRITE(6,1050) L,((XPROB(I,J),J=1,100),I=1,NCHAN)
      L=6HXPTOT
      WRITE(6,1050) L,(XPTOT(I),I=1,100)
      L=6HPBTOT
      WRITE(6,1050) L,(PBTOT(I),I=1,100)
      L=6HELOSS
      WRITE(6,1050) L,(ELOSS(I),I=1,20)
C
C               SET ELECTRON ENERGY
C
      EVT=ENGZER
      VEZ=CETOV*SQRT(EVT)
C
C               INITIAL TIME
C
      T=0.
      NCY=0
      NEE=0
      NEM=0
      NEL=0
      NELU=0
      NCYOLD=0
      NION=0
      NCYION=0
      NWRITE=0
```

```
C
C          WRITE INITIAL VALUES
C
      WRITE(6,VALUES)
C
C          CHOOSE ELECTRON VELOCITY
C
C               RANDOM UNIT VECTOR
C
      KRND=1
  200 RND=URAND(Q)
      R(1)=RND+RND-1.
      WW=SQRT(1.-R(1)**2)
      RND=TWOPI*URAND(Q)
      VV=COS(RND)
      R(2)=WW*VV
      VV=SQRT(1.-VV**2)
      IF(RND.GT.PI) VV=-VV
      R(3)=WW*VV
      GO TO (210,440,460,490),KRND
C
C               SET ELECTRON VECTOR
C
  210 DO 220 I=1,3
  220 VZ(I)=VEZ*R(I)
C
C               ELECTRON PROBABILITIES
C
  250 XPBELC=EPCON*DT*VEZ*TEV
      PBELC=EXP(-XPBELC)
      LOGIC = .FALSE.
      IF(ELECTS-1.)260,260,300
  260 XPBELC=0.
      PBELC=1.
      LOGIC = .TRUE.
C
```

LIRSEM Listing (Cont)

```
C          BASIC MONTE CARLO CYCLE
C
C
C               ADVANCE TIME
C
  300 IF(NCY.GE.NWRITE) GO TO 600
      IF(LOGIC) GO TO 700
  305 NCY=NCY+1
      NSIN=MOD(NCY,NDT)+1
C
C
C               ADD LASER INDUCED VELOCITY
C
      VT(1)=VZ(1)+VF(NSIN)
      VT(2)=VZ(2)
      VT(3)=VZ(3)
C
C
C               NEW KINETIC ENERGY
C
      EVT=0.
      DO 310 I=1,3
  310 EVT=EVT+VT(I)**2
      EVT=CVTOE*EVT
      JE=MIN0(IFIX(1.+5.*EVT),100)
C
C
C               SELECT CHANNEL
C
      XTPB=XPBELC+XPTOT(JE)
      ICH=0
      IF(XTPB-APROX) 320,320,340
  320 IF(XTPB-URAND(Q))300,330,330
  330 IF(XPBELC-XTPB*URAND(Q))350,350,390
  340 IF(PBELC-URAND(Q))390,345,345
  345 IF(PBTOT(JE)-URAND(Q))350,300,300
C
C
C               NON-ELECTRON CHANNEL
C
  350 RND=XPTOT(JE)*URAND(Q)
```

LIRSEM Listing (Cont)

```
      XPB=0.
      DO 370 I=1,NCHAN
      XPB=XPB+XPROB(I,JE)
      IF(XPB-RND) 370,360,360
  360 ICH=I
      GO TO 390
  370 CONTINUE
      ICH=NCHAN
C
C                 MODIFY DISTRIBUTION
C
  390 TE(JE)=TE(JE)+DT*(NCY-NCYOLD)
      NCYOLD=NCY
C
C                 NEW TEMPERATURE
C
      SUM=0.
      TEV=0.
      DO 400 I=1,100
      SUM=SUM+TE(I)
  400 TEV=TEV+TE(I)*I
      TEV=.2*TEV/SUM-.1
C
C                 BRANCH ON CODE NUMBER
C
      IF(ICH.LT.1) GO TO 410
      IF(ICH.EQ.1) GO TO 530
      IF(ICH.NE.2) GO TO 480
      GO TO 540
C
C                 ELECTRON-ELECTRON SCATTERING CHANNEL
C
C                      CHOOSE SECOND ELECTRON
C
  410 TSUM=0.
      NEE=NEE+1
```

LIRSEM Listing (Cont)

```
      RND=SUM*URAND(Q)
      DO 420 JE2=1,100
      TSUM=TSUM+TE(JE2)
      IF(TSUM-RND)420,430,430
  420 CONTINUE
C
C                     RANDOM VELOCITY
C
  430 KRND=2
      GO TO 200
  440 DO 445 I=1,3
  445 VZ2(I)=VE(JE2)*R(I)
C
C                     COMPTON SCATTERED ELECTRON VELOCITY
C                          CENTER OF MASS VELOCITY
C
      SUM=0.
      DO 450 I=1,3
      W(I)=.5*(VZ(I)-VZ2(I))
  450 SUM=SUM+W(I)**2
      WLGTH=SQRT(SUM)
C
C                          SCATTERING ANGLE SIN AND COS
C
      AA=CAA*URAND(Q)*TEV/ELECTS
      BB=CBB/SUM**2
      UC=(AA-BB)/(AA+BB)
      IF(UC-ELNO) 455,455,300
  455 US=SQRT(1.-UC**2)
      NELU=NELU+1
      UW=US/WLGTH
C
C                          RANDOM UNIT VECTOR
C
      KRND=3
      GO TO 200
```

```
C
C                          NEW CM AND LAB VELOCITY
C
  460 SUM=0.
      DO 470 I=1,3
      W(I)=WLGTH*US*R(I)+(UC-UW*R(I)*W(I))*W(I)
  470 VZ(I)=.5*(VZ(I)+VZ2(I))+W(I)
      VEZ=SQRT(VZ(1)**2+VZ(2)**2+VZ(3)**2)
      GO TO 250
C
C                NON-ELECTRON SCATTERING CHANNEL
C
C                     LOSS AND NEW VELOCITY
C
  480 EVT=EVT-ELOSS(ICH)
  485 EVT=AMAX0(0.,EVT)
      VEZ=CETOV*SQRT(EVT)
      KRND=4
      GO TO 200
  490 DO 500 I=1,3
      VT(I)=VEZ*R(I)
  500 VZ(I)=VT(I)
      VZ(1)=VZ(1)-VF(NSIN)
C
C                     COUNT NON-ELECTRON COLLISIONS
C
      NEM=NEM+1
      GO TO 250
C
C                ELASTIC SCATTERING CHANNEL
C
  530 EVT=CELOSS*EVT
      NEL=NEL+1
      GO TO 485
C
C                IONIZATION CHANNEL
```

LIRSEM Listing (Cont)

```
C
  540 NION=NION+1
      NCYION=NCY
      EVT=AMAX1(.5*(EVT-ELOSS(ICH)),0.)
      GO TO 485
C
C          WRITE OUTPUT HISTOGRAM
C
  600 NX=39
      GO TO 800
  601 CONTINUE
      T=NCY*DT
      DO 610 I=1,NX
      IX(I)=5*I-4
      J=2.5*I+1.E-6
      IF(MOD(J,2).EQ.1) GO TO 605
      IY(I)=(TE(J-1)+TE(J)+.5*TE(J+1))/DT+.5
      GO TO 610
  605 IY(I)=(.5*TE(J-2)+TE(J-1)+TE(J))/DT+.5
  610 CONTINUE
      CALL HISTGM(IT,ITX,ITY,IX,IY,NX)
      WRITE(6,1040) T
      RATION=FLOAT(NION)/FLOAT(NCYION)/DT
      RATERR=SQRT(FLOAT(NION))/FLOAT(NCYION)/DT
      WRITE(6,OUTPUT)
      NWRITE=NWRITE+NEDIT
      IF(NWRITE.GT.NQUIT) GO TO 100
      GO TO 305
C
C               NO-ELECTRON CHANNEL CHOICE
C
  700 SUM=0.
      DO 710 I=1,NDT
      EVC=CVTOE*((VZ(1)+VF(I))**2+VZ(2)**2+VZ(3)**2)
      JEC(I)=MIN0(IFIX(1.+5.*EVT),100)
      J=JEC(I)
```

LIRSEM Listing (Cont)

```
  710 SUM=SUM+XPTOT(J)
      RND=ALOG(1./URAND(Q))
      NCY=NCY+IFIX(RND/SUM)
      RND=AMOD(RND,SUM)
      SUM=0.
      DO 720 I=1,NDT
      NCY=NCY+1
      JE=JEC(I)
      SUM=SUM+XPTOT(JE)
      IF(SUM-RND)720,350,350
  720 CONTINUE
      GO TO 350
C
C          OUTPUT TABLES
C
  800 WRITE(6,2000)IT,T
 2000 FORMAT(1H1,80A1//1X,13HTIME (SEC) = ,1E10.4/)
      KH=1
      GO TO 835
  810 WRITE(6,2010)
 2010 FORMAT(43X,38HELECTRON ENERGY DISTRIBUTIION (SEC/EV)/)
      WRITE(6,2020) (H(I),I=1,10)
 2020 FORMAT(1X,10HENERGY(EV),10F10.1/)
      DO 830 I=0,9
      EG=2*I
      DO 820 J=1,10
  820 H(J)=5.*TE(10*I+J)
  830 WRITE(6,2030) EG,(H(J),J=1,10)
 2030 FORMAT(1X,1F9.1,1X,10E10.3)
  835 KH=2
      H(1)=.1
      DO 840 I=2,10
  840 H(I)=H(I-1)+.2
      GO TO (810,845),KH
  845 WRITE(6,2040)
 2040 FORMAT(//,35X,
```

LIRSEM Listing (Cont)

```
     1 53HNORMALIZED ELECTRON ENERGY DISTRIBUTION (FRACTION/EV) /)
      WRITE(6,2020) (H(I),I=1,10)
      SUM=0.
      DO 850 I=1,100
  850 SUM=SUM+TE(I)
      DO 870 I=0,9
      EG=2*I
      DO 860 J=1,10
  860 H(J)=.5*TE(10*I+J)/SUM
  870 WRITE(6,2030)EG,(H(J),J=1,10)
      WRITE(6,OUTPUT)
      GO TO 601
      END
```

REFERENCES

1. Smith, D. C., "Gas-Breakdown Dependence on Beam Size and Pulse Duration with 10.6 μ Wavelength Radiation," Appl. Phys. Let. 19, 405 (1971).

2. Hull, R. J., Marquet, L. C., and Lencioni, D. E., Optics Research, Lincoln Laboratory, ESD-TR-2 (1971), Lexington, Mass.

3. Zeldovich, Y. B., and Raizer, Y. P., "Cascade Ionization of a Gas by a Light Pulse," Soviet Phys. JETP 20, 772 (1965).

4. Chanin, L. M., Phelps, A. V., Biondi, M. A., "Measurements of the Attachment of Low Energy Electrons to Oxygen Molecules," Phys. Rev., 128, 219, 1962.

5. Schulz, G. J., "Vibrational Excitation of N_2, CO, and H2 by Electron Impact," Phys. Rev., 135, A988, 1964.

6. Engelhardt, A. G., Phelps, A. V., Risk, C. G., "Determination of Momentum Transfer and Inelastic Collision Cross Sections for Electrons in Nitrogen Using Transport Coefficients," Phys. Rev., 135, A1566, 1964.

7. Hake, R. D., Phelps, A. V., "Momentum-Transfer and Inelastic-Collision Cross Sections for Electrons in O_2, CO, and CO_2," Phys. Rev., 158, 70, 1967.

8. Borst, W., "Excitation of Several Important Metastable States of N_2 by Electron Impact," Phys. Rev., A5, No. 2 p. 648 (Feb. 1972).

9. Ajello, J., "Emission Cross Sections of N_2 in the Vacuum Ultraviolet by Electron Impact," Jour. Chem. Phys., 53, No. 3, p. 1156 (Aug. 1970).

References (Cont.)

10. Goldstein, H., Classical Mechanics, p. 84, Addison-Wesley Press, Inc., Cambridge, Mass. (1951).

11. Kroll, N., and Watson, K., "Theoretical Study of Ionization of Air by Intense Laser Pulses," Phys. Rev., A5, No. 4, 1883 (April 1972).

12. Chung, S., and Lin, C., "Excitation of the Electronic States of the Nitrogen Molecule by Electron Impact," Phys. Rev., A6, No. 3, 988 (Sept. 1971).

13. Golden, D., "Low Energy Resonances in e^- -N_2 Total Scattering Cross Sections: The Temporary Formation of N_2^-," Phys. Rev. Lett., 17, No. 16, 847 (17 Oct. 1966).

14. Baraff, G., and Buchsbaum, S., "Anisotropic Electron Distribution and the dc and Microwave Avalanche Breakdown in Hydrogen," Phys. Rev. 130, 1007 (1 May 1963).

15. Abramowitz, M., Stegun, G. A., Handbook of Mathematical Functions, p. 376 (1965) Dover Publications, Inc., N. Y.

16. Canavan, G. H., Proctor, W. A., Nielsen, P. E., and Rockwood, S. D., "CO_2 Laser Air Breakdown Calculations," IEEE J. Quant. Electronics QE8, 564 (1972).

17. Richtmyer, R. D., and Morton, K. W., Difference Methods for Initial-Value Problems, Interscience Publishers, New York (1967), 2nd ed.

18. Chapman, S., and Cowling, T., The Mathematical Theory of Non-Uniform Gases, Cambridge University Press, Chapt. 10 (1958).

19. Jung, C. E., Air Chemistry and Radioactivity, Academic Press, New York (1963).

References (Cont.)

20. American Institute of Physics Handbook, McGraw-Hill, New York (1963), 2nd ed.

21. Wolfe, W., Handbook of Military Infrared Technology, Office of Naval Research, Department of the Navy, Washington, D.C. (1965).

22. Gryvnak, D. A., and Burch, P. E., "Optical and Infrared Properties of Al_2O_3 at Elevated Temperatures," J. Opt. Soc. Amer. 55, 625 (1965).

23. Chang, D. B., Drummond, J. E. and Hall, R. B., "High-Power Laser Radiation Interaction with Quartz," J. Appl. Phys. 41, 4851 (1970).

24. Basov, N. G., et al. "Reduction of Reflection Coefficient for Intense Laser Radiation on Solid Surfaces," Sov. Phys.-Tech. Phys. 13, 1581 (1969).

25. McCarthy, D. E., "The Reflection and Transmission of Infrared Materials: I, Spectra from 2-50 Microns," Applied Optics 2, 591 (1963).

26. Hoss, G. and Salzberg, C. D., "Optical Properties of Silicon Monoxide in the Wavelength Region from 0.24 to 14.0 Microns," J. Opt. Soc. Amer. 44, 181 (1954).

27. Calingaert, Heron and Stair, Trans. Soc. Auto. Eng. 39, 448 (1936) as reported in American Institute of Physics Handbook, 2nd ed. (1963) p. (6-66).

28. Ubbelohde, A. R. and Lewis, F. A., Graphite and Its Crystal Components, Oxford Press, 1960, Clarendon Press.

29. Harris, Louis, The Optical Properties of Metal Blocks and Carbon Blocks, MIT, Cambridge, Mass. (December, 1967), p. 91.

30. Nielsen, P. E. and Canavan, G. H., Private Communications, Air Force Weapons Laboratory, LRD-71-1 (1971) Kirtland AFB, N. M. pg. 273.

31. Brodie, R. N. and Hormuth, J. E., The PUFF 66 and P PUFF 66 Computer Programs, AFWF-TR-66-48, Air Force Weapons Laboratory, Kirtland AFB, New Mexico, May 1966. (Private Communications)

32. Kohn, B. J., Compilation of Hugonoit Equations of State, AFWL-TR-69-38, Air Force Weapons Laboratory, Kirtland AFB, New Mexico, April 1969.

33. Zavitsanos, P. D. "Mass Spectrometric Analysis of Carbon Species Generated by Laser Evaporation," Carbon 6, 731 (1968).

34. Gilmore, F. R. Equilibrium Composition and Thermodynamic Properties of Air to 24000°K, Rand Corp., Santa Monica, Calif., RM1543, Aug. 1955. (Unpublished, Private Communication)

35. Zel'dovich, Y. B. and Raizer, Y. P., Physics of Shock Waves and High Temperature Hydrodynamic Phenomena, Acad. Press, New York (1966).

36. Vedeneyev, V. I. et al., Band Energies Ionization Potentials and Electron Affinities, St. Martin's Press, New York (1966).

37. Sears, F. W. An Introduction to Thermodynamics, The Kinetic Theory of Gases, and Statistical Mechanics, Addison-Wesley Publishing, Reading, Mass. (1953), 2nd ed.

38. Peierls, R. E., Quantum Theory of Solids 1st ed., Oxford at the Clarendon Press, p. 97 (1955).

39. Ready, John F., Effects of High-Power Laser Radiation, Acad. Press, N. Y. p. 207 (1971).

40. Personal communication with K. Brueckner, KMS Industries, Ann Arbor, Mich.

41. Tolman, R. C. The Principles of Statistical Mechanics, Oxford University Press (1938).

42. Rapp, D., Englander-Golden, P., "Total Cross Sections for Ionization and Attachment in Gases by Electron Impact. I. Positive Ionization," Jour. Chem. Phys., 43, 1464, 1965.

43. Rapp, D., Briglia, D. D., "Total Cross Sections for Ionization and Attachment in Gases by Electron Impact. II. Negative-Ion Formation," Jour. Chem. Phys., 43, 1480, 1965.

44. Spitzer, L. Physics of Fully Ionized Gases, Interscience, N. Y. (1967).

45. Hammersley, J. M., Handscomb, D. C., Monte Carlo Methods, John Wiley & Sons, N. Y. (1964).